

Constructions and Combinatorial Problems in Design of Experiments

by

Damaraju Raghavarao

*Professor and Chairperson
Department of Statistics
Temple University, Philadelphia*

*Formerly Professor and Head
Department of Mathematics and Statistics
Punjab Agricultural University, Ludhiana, India*

DOVER PUBLICATIONS, INC., *New York*

Contents

Chapter 1 Complete Sets of Mutually Orthogonal Latin Squares 1

1.1. Latin squares and orthogonal Latin squares, 1—1.2. Upper bound for the number of orthogonal Latin squares, 2—1.3. Construction of complete sets of mols, 3—1.4. Connection between complete sets of mols and $PG(2, s)$, 5—1.5. Nonexistence of complete sets of mols, 6—1.6. Embedding to a complete set of mols, 7—References, 8—Bibliography, 8.

Chapter 2 Orthogonal Arrays 9

2.1. Introduction and definition, 9—2.2. Maximum number of constraints, 11—2.3. Use of projective geometry in the construction of orthogonal arrays, 19—2.4. Construction of orthogonal arrays of index unity, 21—2.5. Method of differences in the construction of orthogonal arrays, 22—2.6. Orthogonal arrays $(2s^s, 2(s^s - 1)/(s - 1) - 1, s, 2)$, 26—2.7. Product of orthogonal arrays, 28—2.8. Embedding of orthogonal arrays, 28—2.9. Partially balanced arrays, 29—References, 30—Bibliography, 31.

Chapter 3 Pairwise Balanced Designs and Mutually Orthogonal Latin Squares 32

3.1. Introduction, 32—3.2. MacNeish–Mann theorem, 34—3.3. Pairwise balanced designs, 36—3.4. Application of pairwise balanced designs in the construction of mols, 37—3.5. Use of the method of differences in the construction of mols, 41—3.6. Falsity of Euler's conjecture, 43—3.7. Greatest lower bound on the number of mols of order $s \leq 100$, 43—References, 45—Bibliography, 46.

Chapter 4 General Properties of Incomplete Block Designs 48

4.1. Introduction, 48—4.2. Connectedness, 49—4.3. Balancing in connected and disconnected designs, 51—4.4. Kronecker-product designs, 56—4.5.

PBIB designs, 173—8.17. Concluding remarks, 175—References, 175—Bibliography, 181.

Chapter 9 Graph Theory and Partial Geometries 183

9.1. Introduction, 183—9.2. Some results on graph theory, 183—9.3. Line graph of BIB designs, 186—9.4. Connection of PBIB designs and their association schemes with graph theory, 187—9.5. Finite nets, 190—9.6. Partial geometry (r, k, t) , 192—9.7. Extended partial geometry, 195—References, 197—Bibliography, 198.

Chapter 10 Duals of Incomplete Block Designs 199

10.1. Introduction, 199—10.2. Dual of an incomplete block design to be a specified design, 199—10.3. Duals of asymmetrical BIB designs with $\lambda = 1$ or $\lambda = 2$, 204—10.4. Duals of some PBIB designs, 206—10.5. Linked block designs, 209—References, 211—Bibliography, 211.

Chapter 11 Symmetrical Unequal-Block Arrangements with Two Unequal Block Sizes 212

11.1. Introduction, 212—11.2. Combinatorial problems in SUB arrangements, 213—11.3. Construction of SUB arrangements from known PBIB designs, 214—11.4. Construction of SUB arrangements from known BIB and PBIB designs, 218—References, 221—Bibliography, 221.

Chapter 12 Nonexistence of Incomplete Block Designs 222

12.1. Introduction, 222—12.2. Nonexistence of symmetrical BIB designs, 222—12.3. Nonexistence of affine α -resolvable BIB designs, 224—12.4. Nonexistence of symmetrical, regular PBIB designs, 226—12.5. Nonexistence of certain asymmetrical PBIB designs, 232—12.6. Nonexistence of affine α -resolvable PBIB designs with two associate classes, 236—12.7. Nonexistence of BIB designs in the useful range, 240—References, 241—Bibliography, 241.

Chapter 13 Confounding in Symmetrical Factorial Experiments 243

13.1. Introduction, 243—13.2. Confounding in 2^n experiments, 245—13.3. Confounding in s^n experiments through pencils, 247—13.4. Principle of generalized interaction, 249—13.5. Confounding with the help of pseudo-factors, 249—13.6. Alternative method of confounding, 250—13.7. Packing problem, 252—13.8. Balancing in factorial experiments, 254—13.9. Balanced factorial experiments, 257—References, 258—Bibliography, 259.

Chapter 14 Confounding in Asymmetrical Factorial Experiments 260

14.1. Introduction, 260—14.2. Total relative loss of information in confounded experiments, 261—14.3. Confounded asymmetrical factorial experiments as PBIB designs, 262—14.4. Confounding in asymmetrical factorials where the levels of factors are different powers of the same prime, 264—14.5. Confounding in $3^m \times 2^n$ experiments, 265—14.6. Confounding in $r \times s^m$ experiments in blocks of rs^{m-1} plots, 266—14.7. Use of finite geometries in the construction of confounded asymmetrical designs, 268—14.8. Confounded asymmetrical $3^m \times 2^n$ experiments in blocks of 2^k plots, 270—References, 270—Bibliography, 271.

Chapter 15 Fractional Replication 272

15.1. Introduction, 272—15.2. $1/2^k$ replicate of 2^n factorial experiments, 273—15.3. $1/s^k$ replicate of s^n factorial experiments, 274—15.4. Fractional replicate plans with the help of mols, 276—15.5. Use of Hadamard matrices in the construction of fractional replicate plans, 277—15.6. Use of orthogonal arrays in the construction of fractional replicate plans, 277—15.7. Special types of 2^n fractional replicate plans, 277—15.8. Irregular fractions of 2^n designs, 280—15.9. Fractional plans for asymmetrical factorial experiments, 281—15.10. Nonorthogonal fractional plans, 286—References, 287—Bibliography, 289.

Chapter 16 Rotatable Designs 291

16.1. Introduction and definition, 291—16.2. Simple geometrical designs, 293—16.3. Second- and third-order rotatable designs with the help of a transformation group, 293—16.4. Second-order rotatable designs in k dimensions from second-order rotatable designs in $k-1$ dimensions, 296—16.5. Use of BIB Designs and symmetrical unequal-block arrangements in the construction of second-order rotatable designs, 298—16.6. Construction of third-order rotatable designs through incomplete block designs, 301—16.7. Generalizations of rotatable designs, 302—References, 303—Bibliography, 303.

Chapter 17 Weighing Designs 305

17.1. Introduction, 305.

A. Chemical-Balance Weighing Designs 307

17.2. The model, 307—17.3 Variance limit of estimated weights, 308—17.4. Hadamard matrices and optimum designs, 309—17.5. Efficiency criteria, 315—17.6. Best weighing designs when n is odd or $n \equiv 2 \pmod{4}$, 316.

B. Spring-Balance Weighing Designs 319

17.7. Variance limits of estimated weights and optimum weighing designs, 319.

C. Singular Weighing Designs 321

17.8. Need for considering singular weighing designs, 321—17.9. Estimable parametric functions of the weights for singular weighing designs, 322—17.10 A simple way of taking $p - r$ additional weighings for a singular weighing design, 324—17.11. Best way of taking the additional weighing when $r = p - 1$, 327—17.12. Some results in the general case, 330—References, 334—Bibliography, 336.

Appendix A Mathematics for Statisticians

337

A.1. Mathematical systems, 337—A.2. Galois fields, 338—A.3. Vector algebra, 341—A.4. Matrix algebra, 344—A.5. Quadratic forms, 353—A.6. The Legendre symbol, the Hilbert norm residue symbol and the Hasse-Minkowski invariant, 354—A.7. Finite geometries, 357—References, 360—Bibliography, 360.

Appendix B Statistics for Mathematicians

361

B.1. Probability, 361—B.2. Random variable and distribution functions, 362—B.3. Parameters of the distribution function, 363—B.4. Multidimensional random variable, 363—B.5. Normal distribution and related distributions, 365—B.6. Theory of linear estimation and tests of linear hypothesis, 366—B.7. Block designs, 368—Reference, 370—Bibliography, 370.

Miscellaneous Exercises

371

Index

379