

FAMOUS PROBLEMS

AND OTHER MONOGRAPHS

FAMOUS PROBLEMS OF ELEMENTARY GEOMETRY

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FROM DETERMINANT TO TENSOR

BY W. F. SHEPPARD

INTRODUCTION TO COMBINATORY ANALYSIS

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THREE LECTURES ON FERMAT'S LAST THEOREM

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CONTENTS.

INTRODUCTION.

	PAGE
PRACTICAL AND THEORETICAL CONSTRUCTIONS	2
STATEMENT OF THE PROBLEM IN ALGEBRAIC FORM	3

PART I.

The Possibility of the Construction of Algebraic Expressions.

CHAPTER I. ALGEBRAIC EQUATIONS SOLVABLE BY SQUARE ROOTS.

1-4. Structure of the expression x to be constructed	5
5, 6. Normal form of x	6
7, 8. Conjugate values	7
9. The corresponding equation $F(x) = 0$	8
10. Other rational equations $f(x) = 0$	8
11, 12. The irreducible equation $\phi(x) = 0$	10
13, 14. The degree of the irreducible equation a power of 2	11

CHAPTER II. THE DELIAN PROBLEM AND THE TRISECTION OF THE ANGLE.

1. The impossibility of solving the Delian problem with straight edge and compasses	13
2. The general equation $x^3 = \lambda$	13
3. The impossibility of trisecting an angle with straight edge and compasses	14

CHAPTER III. THE DIVISION OF THE CIRCLE INTO EQUAL PARTS.

1. History of the problem	16
2-4. Gauss's prime numbers	17
5. The cyclotomic equation	19
6. Gauss's Lemma	19
7, 8. The irreducibility of the cyclotomic equation	21

CONTENTS.

CHAPTER IV. THE CONSTRUCTION OF THE REGULAR POLYGON OF 17 SIDES.

	PAGE
1. Algebraic statement of the problem	24
2-4. The periods formed from the roots	25
5, 6. The quadratic equations satisfied by the periods	27
7. Historical account of constructions with straight edge and compasses	32
8, 9. Von Staudt's construction of the regular polygon of 17 sides	34

CHAPTER V. GENERAL CONSIDERATIONS ON ALGEBRAIC CONSTRUCTIONS.

1. Paper folding	42
2. The conic sections	42
3. The Cissoid of Diocles	44
4. The Conchoid of Nicomedes	45
5. Mechanical devices	47

PART II.

Transcendental Numbers and the Quadrature of the Circle.

CHAPTER I. CANTOR'S DEMONSTRATION OF THE EXISTENCE OF TRANSCENDENTAL NUMBERS.

1. Definition of algebraic and of transcendental numbers	49
2. Arrangement of algebraic numbers according to <i>height</i>	50
3. Demonstration of the existence of transcendental numbers	53

CHAPTER II. HISTORICAL SURVEY OF THE ATTEMPTS AT THE COM- PUTATION AND CONSTRUCTION OF π .

1. The empirical stage	56
2. The Greek mathematicians	56
3. Modern analysis from 1670 to 1770	58
4, 5. Revival of critical rigor since 1770	59

CHAPTER III. THE TRANSCENDENCE OF THE NUMBER e .

1. Outline of the demonstration	61
2. The symbol h^* and the function $\phi(x)$	62
3. Hermite's Theorem	65

CONTENTS.

CHAPTER IV. THE TRANSCENDENCE OF THE NUMBER π .

	PAGE
1. Outline of the demonstration	68
2. The function $\psi(x)$	70
3. Lindemann's Theorem	73
4. Lindemann's Corollary	74
5. The transcendence of π	76
6. The transcendence of $y = e^x$	77
7. The transcendence of $y = \sin^{-1}x$	77

CHAPTER V. THE INTEGRAPH AND THE GEOMETRIC CONSTRUCTION OF π .

1. The impossibility of the quadrature of the circle with straight edge and compasses	78
2. Principle of the integraph	78
3. Geometric construction of π	79

NOTES	81
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