Vladimir Kozlov Vladimir Maz'ya

Theory of a Higher-Order Sturm-Liouville Equation

Springer

Table of Contents

Introduction V

1. Basic Equation with Constant Coefficients 1
- 1.1 Introduction 1
- 1.2 Green's Function for $M(\partial_t)$ on $\mathbb{R}$ 2
- 1.3 Necessary and Sufficient Condition for Solvability 4
- 1.4 Necessary and Sufficient Condition for Power-Exponential Asymptotics 8
- 1.5 A Larger Class of Ordinary Differential Equations 12

2. The Operator $M(\partial_t)$ on a Semiaxis and an Interval 15
- 2.1 Introduction 15
- 2.2 Properties of Green's Function 15
- 2.3 Unique Solvability of the Boundary Value Problem on a Semiaxis 17
- 2.4 Asymptotics at Infinity 18
- 2.5 Unique Solvability of the Boundary Value Problem on a Finite Interval 20
- 2.6 Positivity of Poisson's Functions 21
- 2.7 Green's Function for the Boundary Value Problem on an Interval 21
- 2.8 Positivity of Green's Function 23

3. The Operator $M(\partial_t) - \omega_0$ with Constant ω_0 25
- 3.1 Introduction 25
- 3.2 Roots of the Characteristic Equation 26
- 3.3 Green's Function for $M(\partial_t) - \omega_0$ on $\mathbb{R}$ 28
- 3.4 Uniqueness and Solvability 31
- 3.5 Stable Estimate 34

4. Green's Function for the Operator $M(\partial_t) - \omega(t)$ 37
- 4.1 Properties of Green's Function 37
- 4.2 Monotonicity Properties of Green's Function with Respect to ω 39

4.3 Multiplicative Estimate for Green's Function and its Derivativ
4.4 Other Estimates for Green's Function 43

5. Uniqueness and Solvability Properties of the Operator $M(\partial_t) - \omega(t)$ 47
5.1 Introduction 47
5.2 Existence Theorem and Properties of Solutions 48
5.3 The Equation on a Bounded Interval 52
5.4 Uniqueness 55
5.5 Some Corollaries of the Uniqueness Theorem 61
5.6 The Boundary value Problem on a Semiaxis 63
5.7 Representation for g_ω 67
5.8 Properties of g_ω and Uniqueness Theorem when $m_+ = 1$ or $m_- = 1$ 72
5.9 The Space $X_+(\omega)$ for $m_+ = 2$ 73
5.10 Green's Function and Uniqueness Theorem when $m_+ = 2$ 77
5.11 The Space $X_-(\omega)$ for $m_- = 2$ 78

6. Properties of $M(\partial_t) - \omega(t)$ under Various Assumptions about $\omega(t)$ 81
6.1 Introduction 81
6.2 The Case of Bounded Coefficient ω 84
6.3 The Case of Summable ω 86
6.4 The Case $m_+ = m_- = 1$ 90
6.5 Uniqueness and Solvability in the Case $m_+ = m_- = 1$ 93
6.6 Power Perturbation (Estimates for g_ω in the Case $\alpha < m_+$) 94
6.7 Power Perturbation (Estimates for g_ω in the Case $\alpha = m_+$) 98
6.8 Power Perturbation (Uniqueness and Solvability) 100
6.9 Estimates for g_ω when ω 'Dominates' either t^{-m_+} or t^{-m_-} for Large Positive t 101
6.10 Uniqueness and Existence Theorems in the Case $(\omega^{-1/m_-})' \to 0$ 110

7. Asymptotics of Solutions at Infinity 111
7.1 Estimates for Green's Function g_ω under the Assumption of Weighted Summability of $\omega(t)$ 111
7.1.1 Statement of the Result 111
7.1.2 Proof of Theorem 7.1.1 112
7.1.3 Remarks and Corollaries 116
7.2 Auxiliary Existence Assertions 118
7.3 Asymptotics of Solutions of the Homogeneous Equation 121
7.4 Necessary and Sufficient Condition for Asymptotics 124

A. Application to Ordinary Differential Equations with Operator Coefficients 127
A.1 Comparison Principle for Equations with Constant Operator Coefficients 127
A.2 Comparison Principle for Equations with Variable Operator Coefficients 130
A.3 Some Special Cases 132
A.3.1 The Case $m_+ = m_- = 1$ 132
A.3.2 Power Perturbations 133
A.4 Power-exponential Asymptotics of Solutions to the Operator Equation with Variable Coefficients 134

References 137

Index 139