

# Ergebnisse der Mathematik und ihrer Grenzgebiete

3. Folge - Band 20

A Series of Modern Surveys in Mathematics

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