

PROBLEM BOOK

**IN THE
THEORY OF FUNCTIONS**

Volume II

Problems in the Advanced Theory of Functions

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¹⁾ See the Foreword.

FOREWORD

This translation is of the fourth (1949) edition of K. Knopp's *Aufgabensammlung zur Funktionentheorie, II. Teil*.

References to the first volume of this Problem Book are indicated by writing "I" together with chapter, page or paragraph, and problem numbers; references to the present second volume contain only page or paragraph, and problem numbers. This second volume adheres strictly to the little function-theoretical volumes put out (except the last one) by Dover Publications. These works are quoted as follows:

- I = *Problem Book in the Theory of Functions*, vol. I: *Problems in the Elementary Theory of Functions*, translated by L. Bers, 1948.
- Elem. = Knopp, *Elements of the Theory of Functions*, translated by F. Bagemihl, 1952.
- K I = Knopp, *Theory of Functions*, Part I, translated by F. Bagemihl, 1945.
- K II = Knopp, *Theory of Functions*, Part II, translated by F. Bagemihl, 1947.
- B = Bieberbach, *Einführung in die konforme Abbildung*, 4th. ed., Berlin, 1949.

The present volume again contains mainly exercises which remain essentially within the range of ideas appearing in the volumes just mentioned. Only within this frame, and not in any absolute sense, is the division into elementary and advanced function theory intended. The present problems are based mostly on the last chapters of K I, as well as on K II and B. For using the book, the prefatory remarks to I are pertinent. Corresponding to the higher level now, the asterisk (*) is used more sparingly to denote the harder problems.