Mathematical Surveys and Monographs Volume 5 I

> Fine Regularity of Solutions of Elliptic Partial Differential Equations

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