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Continuous Martingales and Brownian Motion

Corrected Third Printing of the Third Edition

With 8 Figures

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