

GREEN'S FUNCTIONS

G. F. ROACH

Professor of Mathematics, University of Strathclyde

SECOND EDITION

 CAMBRIDGE
UNIVERSITY PRESS

CONTENTS

<i>Preface to the First Edition</i>	xi
<i>Preface to the Second Edition</i>	xiv
CHAPTER 1 THE CONCEPT OF A GREEN'S FUNCTION	1
CHAPTER 2 VECTOR SPACES AND LINEAR TRANSFORMATIONS	9
2.1 Vector Spaces	9
2.2 Linearly Independent Vectors	16
2.3 Orthonormal Vectors	20
2.4 Linear Transformations	24
CHAPTER 3 SYSTEMS OF FINITE DIMENSION	31
3.1 Matrices and Linear Transformations	31
3.2 Change of Basis	36
3.3 Eigenvalues and Eigenvectors	38
3.4 Symmetric Operators	51
3.5 Bounded Operators	55
3.6 Positive Definite Operators	59
CHAPTER 4 CONTINUOUS FUNCTIONS	61
4.1 Limiting Processes	61
4.2 Continuous Functions	65
CHAPTER 5 INTEGRAL OPERATORS	79
5.1 The Kernel of an Integral Operator	79
5.2 Symmetric Integral Transformations	83
5.3 Separable Kernels	85
5.4 Eigenvalues of a Symmetric Integral Operator	91
5.5 Expansion Theorems for Integral Transformations	99
CHAPTER 6 GENERALIZED FOURIER SERIES AND COMPLETE VECTOR SPACES	112
6.1 Generalized Fourier Series	112
6.2 Approximation Theorem	121
6.3 Complete Vector Spaces	127

CHAPTER 7 DIFFERENTIAL OPERATORS	141
7.1 Introduction	141
7.2 Inverse Operators and the δ -function	141
7.3 The Domain of a Linear Differential Operator	152
7.4 Adjoint Differential Operators	154
7.5 Self-Adjoint Second-Order Differential Operators	157
7.6 Non-Homogeneous Problems and Symbolic Operators	159
7.7 Green's Functions and Second-Order Differential Operators	163
7.8 The Problem of Eigenfunctions	177
7.9 Green's Functions and the Adjoint Operator	181
7.10 Spectral Representation and Green's Functions	182
CHAPTER 8 INTEGRAL EQUATIONS	187
8.1 Classification of Integral Equations	187
8.2 Method of Successive Approximations	188
8.3 The Fredholm Alternative	195
8.4 Symmetric Integral Equations	206
8.5 Equivalence of Integral and Differential Equations	210
CHAPTER 9 GREEN'S FUNCTIONS IN HIGHER-DIMENSIONAL SPACES	213
9.1 Introduction	213
9.2 Partial Differential Operators and δ -functions	215
9.3 Green's Identities	224
9.4 Fundamental Solutions	227
9.5 Self-Adjoint Elliptic Equations (The Dirichlet Problem)	237
9.6 Self-Adjoint Elliptic Equations (The Neumann Problem)	243
9.7 Parabolic Equations	248
9.8 Hyperbolic Equations	251
9.9 Worked Examples	256
CHAPTER 10 CALCULATION OF PARTICULAR GREEN'S FUNCTIONS	274
10.1 Method of Images	274
10.2 Generalized Green's Function	278
10.3 Mixed Problems	287
CHAPTER 11 APPROXIMATE GREEN'S FUNCTIONS	291
11.1 Introduction	291
11.2 Fundamental Solutions	292
11.3 Generalized Potentials	295
11.4 A Representation Theorem	300
11.5 Choice of Approximate Kernel	302

APPENDIX A	SUMMARY OF THE GREEN'S FUNCTION METHOD	304
A1	Green's Function Method for Ordinary Differential Equations	304
A2	Green's Function Method for Partial Differential Equations	305
APPENDIX B	OPERATORS AND EXPRESSIONS	307
APPENDIX C	THE LEBESGUE INTEGRAL	312
APPENDIX D	DISTRIBUTIONS	316
BIBLIOGRAPHY		319
	Chapter References	321
INDEX		323