

---

# **Chaotic dynamics**

## **an introduction**

---

**SECOND EDITION**

**GREGORY L. BAKER**

*Academy of the New Church College  
Beau Atkin, Pennsylvania*

**AND**

**JERRY P. GOLLUB**

*Haverford College, Haverford, Pennsylvania and  
University of Pennsylvania, Philadelphia, Pennsylvania*



**CAMBRIDGE**  
UNIVERSITY PRESS

---

# Contents

---

<i>Preface</i>	page xi	
<i>Acknowledgments</i>	xiv	
<b>CHAPTER ONE</b>	<b>Introduction</b>	<b>1</b>
<b>CHAPTER TWO</b>	<b>Some helpful tools</b>	<b>7</b>
2.1	Phase space	7
2.2	Poincaré section	21
2.3	Spectral analysis of time series	27
Problems		35
<b>CHAPTER THREE</b>	<b>Visualization of the pendulum's dynamics</b>	<b>39</b>
3.1	Sensitivity to initial conditions	41
3.2	Phase diagrams and Poincaré sections	43
3.3	Time series and power spectra	59
3.4	Basins of attraction	59
3.5	Bifurcation diagrams	66
Problems and simulations		72
<b>CHAPTER FOUR</b>	<b>Toward an understanding of chaos</b>	<b>74</b>
4.1	The logistic map	76
4.1.1	Period doubling	77
4.1.2	The periodic windows	81
4.1.3	Lyapunov exponents	84
4.1.4	Entropy	86
4.1.5	Stretching and folding	88
4.2	The circle map	89
4.3	The horseshoe map	96

---

4.4 Application to the pendulum	100
Problems	105
<b>CHAPTER FIVE The characterization of chaotic attractors</b>	109
5.1 Dimension	110
5.2 Lyapunov exponents	119
5.3 Lyapunov exponents and dimension	123
5.4 Information change and Lyapunov exponents	126
Problems	129
<b>CHAPTER SIX Experimental characterization, prediction, and modification of chaotic states</b>	133
6.1 Characterization of chaotic states	133
6.1.1 Experiment and simulation	135
6.1.2 Reconstruction of the attractor	137
6.1.3 Time-delay coordinates	139
6.1.4 Choosing the time delay	143
6.1.5 Embedding dimension and attractor dimension	145
6.1.6 Lyapunov exponents	150
6.1.7 Summary	152
6.2 Prediction of chaotic states	152
6.2.1 Method of analogues	153
6.2.2 Linear approximation method	156
6.3 Modification of chaotic states	159
6.4 Conclusion	163
Problems	164
<b>CHAPTER SEVEN Chaos broadly applied</b>	166
7.1 Chaos in lasers	166
7.2 Chaotic chemical reactions	168
7.3 Chaos in fluid dynamics	170
7.4 Spatio-temporal chaos in fluids	172
7.4.1 Spatio-temporal chaos in thermal convection	173
7.4.2 Spatio-temporal chaos on a rotating fluid film	176
7.5 Spatio-temporal intermittency in model equations	178
7.6 Strong turbulence	179
7.7 Chaotic mixing in fluids	180
7.8 Complex dynamics of interfacial growth: artificial snowflakes	181
7.9 Chaos in earthquake dynamics	184
7.10 Chaos and quantum physics	185

7.11 Foundations of statistical mechanics	187
7.12 Conclusion	189
<i>Further reading</i>	190
<i>Appendix A Numerical integration –</i>	
<i>Runge–Kutta method</i>	193
<i>Appendix B Computer program listings</i>	196
<i>Appendix C Solutions to selected problems</i>	242
<i>References</i>	246
<i>Index</i>	253
<i>Diskette order information</i>	256