

*Theory of Recursive Functions
and Effective Computability*

Hartley Rogers, Jr.

The MIT Press
Cambridge, Massachusetts
London, England

Contents

<i>Preface to paperback edition (1987)</i>	vii
<i>Introduction: Prerequisites and Notation</i>	xv
<i>Emendations</i>	xx
Chapter 1 RECURSIVE FUNCTIONS	1
§1.1 The informal notion of algorithm	1
§1.2 An example: the primitive recursive functions	5
§1.3 Extensionality	9
§1.4 Diagonalization	10
§1.5 Formal characterization	11
§1.6 The Basic Result	18
§1.7 Church's Thesis	20
§1.8 Gödel numbers, universality, $s\text{-}m\text{-}n$ theorem	21
§1.9 The halting problem	24
§1.10 Recursiveness	26
Chapter 2 UNSOLVABLE PROBLEMS	32
§2.1 Further examples of recursive unsolvability	32
§2.2 Unsolvable problems in other areas of mathematics	35
§2.3 Existence of certain partial recursive functions	36
§2.4 Historical remarks	38
§2.5 Discussion	39
§2.6 Exercises	40
Chapter 3 PURPOSES; SUMMARY	46
§3.1 Goals of theory	46
§3.2 Emphasis of this book	48
§3.3 Summary	48
Chapter 4 RECURSIVE INVARIANCE	50
§4.1 Invariance under a group	50
§4.2 Recursive permutations	51

§4.3 Recursive invariance	52
§4.4 Resemblance	53
§4.5 Universal partial functions	53
§4.6 Exercises	55
Chapter 5 RECURSIVE AND RECURSIVELY ENUMERABLE SETS	57
§5.1 Definitions	57
§5.2 Basic theorem	60
§5.3 Recursive and recursively enumerable relations; coding of <i>k</i> -tuples	63
§5.4 Projection theorems	66
§5.5 Uniformity	67
§5.6 Finite sets	69
§5.7 Single-valuedness theorem	71
§5.8 Exercises	73
Chapter 6 REDUCIBILITIES	77
§6.1 General introduction	77
§6.2 Exercises	79
Chapter 7 ONE-ONE REDUCIBILITY; MANY-ONE REDUCIBILITY; CREATIVE SETS	89
§7.1 One-one reducibility and many-one reducibility	89
§7.2 Complete sets	82
§7.3 Creative sets	84
§7.4 One-one equivalence and recursive isomorphism	85
§7.5 One-one completeness and many-one completeness	87
§7.6 Cylinders	89
§7.7 Productiveness	90
§7.8 Logic	94
§7.9 Exercises	99
Chapter 8 TRUTH-TABLE REDUCIBILITIES; SIMPLE SETS	105
§8.1 Simple sets	105
§8.2 Immune sets	107
§8.3 Truth-table reducibility	109
§8.4 Truth-table reducibility and many-one reducibility	112
§8.5 Bounded truth-table reducibility	114
§8.6 Structure of degrees	118

§8.7 Other recursively enumerable sets	120
§8.8 Exercises	121
Chapter 9 TURING REDUCIBILITY; HYPERSIMPLE SETS	127
§9.1 An example	127
§9.2 Relative recursiveness	128
§9.3 Relativized theory	134
§9.4 Turing reducibility	137
§9.5 Hypersimple sets; Dekker's theorem	138
§9.6 Turing reducibility and truth-table reducibility; Post's problem	141
§9.7 Enumeration reducibility	145
§9.8 Recursive operators	148
§9.9 Exercises	154
Chapter 10 POST'S PROBLEM; INCOMPLETE SETS	161
§10.1 Constructive approaches	161
§10.2 Friedberg's solution	163
§10.3 Further results and problems	167
§10.4 Inseparable sets of any recursively enumerable degree	170
§10.5 Theories of any recursively enumerable degree	171
§10.6 Exercises	174
Chapter 11 THE RECURSION THEOREM	179
§11.1 Introduction	179
§11.2 The recursion theorem	180
§11.3 Completeness of creative sets; completely productive sets	183
§11.4 Other applications and constructions	185
§11.5 Other forms of the recursion theorem	192
§11.6 Discussion	199
§11.7 Ordinal notations	205
§11.8 Constructive ordinals	211
§11.9 Exercises	213
Chapter 12 RECURSIVELY ENUMERABLE SETS AS A LATTICE	223
§12.1 Lattices of sets	223
§12.2 Decomposition	230
§12.3 Cohesive sets	231
§12.4 Maximal sets	234
§12.5 Subsets of maximal sets	237
§12.6 Almost-finiteness properties	240
§12.7 Exercises	245

Chapter 13 DEGREES OF UNSOLVABILITY	254
§13.1 The jump operation	254
§13.2 Special sets and degrees	262
§13.3 Complete degrees; category and measure	265
§13.4 Ordering of degrees	273
§13.5 Minimal degrees	276
§13.6 Partial degrees	279
§13.7 The Medvedev lattice	282
§13.8 Further results	289
§13.9 Exercises	295
Chapter 14 THE ARITHMETICAL HIERARCHY (PART 1)	301
§14.1 The hierarchy of sets	301
§14.2 Normal forms	305
§14.3 The Tarski-Kuratowski algorithm	307
§14.4 Arithmetical representation	312
§14.5 The strong hierarchy theorem	314
§14.6 Degrees	316
§14.7 Applications to logic	318
§14.8 Computing degrees of unsolvability	323
§14.9 Exercises	331
Chapter 15 THE ARITHMETICAL HIERARCHY (PART 2)	335
§15.1 The hierarchy of sets of sets	335
§15.2 The hierarchy of sets of functions	346
§15.3 Functionals	358
§15.4 Exercises	367
Chapter 16 THE ANALYTICAL HIERARCHY	373
§16.1 The analytical hierarchy	373
§16.2 Analytical representation; applications to logic	384
§16.3 Finite-path trees	392
§16.4 Π_1^1 -sets and Δ_1^1 -sets	397
§16.5 Generalized computability	402
§16.6 Hyperdegrees and the hyperjump; Σ_1^1 -sets and Δ_1^1 -sets	409
§16.7 Basis results and implicit definability	418
§16.8 The hyperarithmetical hierarchy	434
§16.9 Exercises	445
<i>Bibliography</i>	459
<i>Index of Notations</i>	469
<i>Subject Index</i>	473