

The Foundations of Statistics

LEONARD J. SAVAGE

*Late Eugene Higgins Professor of Statistics
Yale University*

SECOND REVISED EDITION

DOVER PUBLICATIONS, INC.
NEW YORK

Contents

Postulates of a personalistic theory of decision	End papers
--	------------

1. INTRODUCTION

1. The role of foundations	1
2. Historical background	1
3. General outline of this book	4

2. PRELIMINARY CONSIDERATIONS ON DECISION IN THE FACE OF UNCERTAINTY

1. Introduction	6
2. The person	7
3. The world, and states of the world	8
4. Events	10
5. Consequences, acts, and decisions	13
6. The simple ordering of acts with respect to preference	17
7. The sure-thing principle	21

3. PERSONAL PROBABILITY

1. Introduction	27
2. Qualitative personal probability	30
3. Quantitative personal probability	33
4. Some mathematical details	40
5. Conditional probability, qualitative and quantitative	43
6. The approach to certainty through experience	46
7. Symmetric sequences of events	50

4. CRITICAL COMMENTS ON PERSONAL PROBABILITY

1. Introduction	56
2. Some shortcomings of the personalistic view	57
3. Connection with other views	60
4. Criticism of other views	60
5. The role of symmetry in probability	63
6. How can science use a personalistic view of probability?	67

5. UTILITY

1. Introduction	69
2. Gambles	70
3. Utility, and preference among gambles	73
4. The extension of utility to more general acts	76
5. Small worlds	82
6. Historical and critical comments on utility	91

6. OBSERVATION	
1. Introduction	105
2. What an observation is	106
3. Multiple observations, and extensions of observations and of sets of acts	111
4. Dominance and admissibility	114
5. Outline of the design of experiments	116
7. PARTITION PROBLEMS	
1. Introduction	120
2. Structure of (twofold) partition problems	121
3. The value of observation	125
4. Extension of observations, and sufficient statistics	128
5. Likelihood ratios	134
6. Repeated observations	140
7. Sequential probability ratio procedures	142
8. Standard form, and absolute comparison between observations	148
8. STATISTICS PROPER	
1. Introduction	154
2. What is statistics proper?	154
3. Multipersonal problems	154
4. The minimax theory	156
9. INTRODUCTION TO THE MINIMAX THEORY	
1. Introduction	158
2. The behavioralistic outlook	159
3. Mixed acts	162
4. Income and loss	163
5. The minimax rule, and the principle of admissibility	164
6. Illustrations of the minimax rule	165
7. Objectivistic motivation of the minimax rule	168
8. Loss as opposed to negative income in the minimax rule	169
10. A PERSONALISTIC REINTERPRETATION OF THE MINIMAX THEORY	
1. Introduction	172
2. A model of group decision	172
3. The group minimax rule, and the group principle of admissibility	173
4. Critique of the group minimax rule	174
11. THE PARALLELISM BETWEEN THE MINIMAX THEORY AND THE THEORY OF TWO-PERSON GAMES	
1. Introduction	178
2. Standard games	178
3. Minimax play	180
4. Parallelism and contrast with the minimax theories	180
12. THE MATHEMATICS OF MINIMAX PROBLEMS	
1. Introduction	184
2. Abstract games	184

3. Bilinear games	186
4. An example of a bilinear game	189
5. Bilinear games exhibiting symmetry	193
13. OBJECTIONS TO THE MINIMAX RULES	
1. Introduction	200
2. A confusion between loss and negative income	200
3. Utility and the minimax rule	201
4. Almost sub-minimax acts	203
5. The minimax rule does not generate a simple ordering	205
14. THE MINIMAX THEORY APPLIED TO OBSERVATIONS	
1. Introduction	208
2. Recapitulation of partition problems	208
3. Sufficient statistics	212
4. Simple dichotomy, an example	212
5. The approach to certainty	214
6. Cost of observation	214
7. Sequential probability ratio procedures	215
8. Randomization	216
9. Mixed acts in statistics	217
15. POINT ESTIMATION	
1. Introduction	220
2. The verbalistic concept of point estimation	221
3. Examples of problems of point estimation	221
4. Criteria that have been proposed for point estimates	223
5. A behavioralistic review of the criteria for point estimation	229
6. A behavioralistic review, continued	234
7. A behavioralistic review, concluded	244
16. TESTING	
1. Introduction	246
2. A theory of testing	247
3. Testing in practice	252
17. INTERVAL ESTIMATION AND RELATED TOPICS	
1. Estimates of the accuracy of estimates	257
2. Interval estimation and confidence intervals	259
3. Tolerance intervals	262
4. Fiducial probability	262
APPENDIX 1. EXPECTED VALUE	263
APPENDIX 2. CONVEX FUNCTIONS	266
APPENDIX 3. BIBLIOGRAPHIC MATERIAL	270
APPENDIX 4. BIBLIOGRAPHIC SUPPLEMENT	283
TECHNICAL SYMBOLS	290
AUTHOR INDEX	301
GENERAL INDEX	305