

L. Ambrosio L. A. Caffarelli Y. Brenier
G. Buttazzo C. Villani

Optimal Transportation and Applications

Lectures given at the
C.I.M.E. Summer School
held in Martina Franca, Italy,
September 2–8, 2001

Editors: L. A. Caffarelli
S. Salsa



Fondazione
C.I.M.E.



Springer

Contents

The Monge-Ampère Equation and Optimal Transportation, an elementary review

<i>Luis Caffarelli</i>	1
1 Optimal Transportation	1
2 The continuous case:	2
3 The dual problem:	3
4 Existence and Uniqueness:	4
5 The potential equation:	6
6 Some remarks on the structure of the equation	7

Optimal Shapes and Masses, and Optimal Transportation Problems

<i>Giuseppe Buttazzo, Luigi De Pascale</i>	11
1 Introduction	11
2 Some classical problems	13
2.1 The isoperimetric problem	13
2.2 The Newton's problem of optimal aerodynamical profiles	14
2.3 Optimal Dirichlet regions	17
2.4 Optimal mixtures of two conductors	19
3 Mass optimization problems	23
4 Optimal transportation problems	29
4.1 The optimal mass transportation problem: Monge and Kantorovich formulations	30
4.2 The PDE formulation of the mass transportation problem	32
5 Relationships between optimal mass and optimal transportation	33
6 Further results and open problems	35
6.1 A vectorial example	35
6.2 A p -Laplacian approximation	37
6.3 Optimization of Dirichlet regions	38
6.4 Optimal transporting distances	40
References	44

Optimal transportation, dissipative PDE's and functional inequalities

<i>Cedric Villani</i>	53
1 Some motivations	54
2 A study of fast trend to equilibrium	55
3 A study of slow trend to equilibrium	64
4 Estimates in a mean-field limit problem	71
5 Otto's differential point of view	80
References	88

Extended Monge-Kantorovich Theory

<i>Yann Brenier</i>	91
1 Abstract	91
2 Generalized geodesics and the Monge-Kantorovich theory	91
2.1 Generalized geodesics	91
2.2 Extension to probability measures	94
2.3 A decomposition result	96
2.4 Relativistic MKT	97
2.5 A relativistic heat equation	98
2.6 Laplace's equation and Moser's lemma revisited	100
3 Generalized Harmonic functions	102
3.1 Classical harmonic functions	102
3.2 Open problems	105
4 Multiphasic MKT	109
5 Generalized extremal surfaces	111
5.1 MKT revisited as a subset of generalized surface theory	113
5.2 Degenerate quadratic cost functions	113
6 Generalized extremal surfaces in $\mathbb{R}^5$ and Electrodynamics	114
6.1 Recovery of the Maxwell equations	115
6.2 Derivation of a set of nonlinear Maxwell equations	116
6.3 An Euler-Maxwell-type system	118
References	120

Existence and stability results in the L^1 theory of optimal transportation

<i>Luigi Ambrosio, Aldo Pratelli</i>	123
1 Introduction	123
2 Notation	129
3 Duality and optimality conditions	130
4 F -convergence and F -asymptotic expansions	135
5 1-dimensional theory	136
6 Transport rays and transport set	138
7 A stability result	146
8 A counterexample	150
9 Appendix: disintegration of measures	152
References	158