Phillip A. Griffiths
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Rational
Homotopy Theory
and Differential
Forms

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I. Basic Concepts

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II. The CW Homology Theorem

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III. The Whitehead and Hurewicz Theorems

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