

Hans-Dieter Alber

Materials with Memory

Initial-Boundary Value Problems
for Constitutive Equations
with Internal Variables



Springer

Contents

1	Introduction	1
2	Initial-Boundary Value Problems for the Inelastic Behavior of Metals	7
2.1	Formulation of the Initial-Boundary Value Problems	7
2.2	Examples of Constitutive Equations	11
3	Constitutive Equations of Monotone Type and Generalized Standard Materials	23
3.1	Energy Estimate and Classes of Constitutive Equations	23
3.2	Uniqueness and Existence for Dynamic and Quasi-Static Problems: Basic Ideas of the Proofs and Results in the Literature	27
3.3	Examples of Constitutive Equations Revisited	31
3.4	A Criterion for Monotone Type	41
4	Existence of Solutions for Constitutive Equations of Monotone Type	45
4.1	Formulation of the Problem as a First Order Evolution Equation . . .	45
4.2	Maximality of the Evolution Operator	48
4.3	Existence for the Dynamic Problem	56
5	Transformation of Interior Variables	57
5.1	Transformation Fields	57
5.2	Properties of the Transformation: Restrictions Imposed by the ε -Independence of the Transformation Field and Invariance under Linear Transformations	60
5.3	The Class of Constitutive Equations Transformable to Monotone Type	65
5.4	The Class of Constitutive Equations Transformable to Gradient Type	67
5.5	The Class of Constitutive Equations Transformable to Monotone- Gradient Type	69
6	Classification Conditions	75
6.1	Transformations which Leave the Class of Pre-Monotone Equations Invariant	76
6.2	Transformation of Pre-Monotone Equations to Gradient Type	87

7	Transformation of Rate Independent Constitutive Equations	99
7.1	Transformation of Constitutive Equations Containing Set-Valued Operators	99
7.2	Transformation to Monotone-Gradient Type	102
7.3	Example 1: One Variable of Isotropic Hardening	106
7.4	Example 2: Several Variables of Isotropic Hardening	111
8	Application of the Theory to Engineering Models	117
8.1	Pre-Monotone Type of the Model	117
8.2	Conditions for Monotone Type of the Model	119
8.3	Pre-Monotone Type Preserving Transformations of the Model	125
8.4	Transformation to Gradient Type	126
8.5	Transformation to Monotone Type and to Monotone-Gradient Type	129
9	Open Problems and Related Results	137
9.1	Transformation Fields Depending on ε and z	137
9.2	History Functionals	138
9.3	Hysteresis Operators. Existence Theory in L^p	139
9.4	Constitutive Equations Defining Continuous Operators in Banach Spaces	142
A	The Second Law of Thermodynamics and the Dissipation Inequality	143
A.1	Consequences of the Second Law for the Constitutive Equations . . .	143
A.2	The Free Energy	149
	Bibliography	153
	Index	165