

Projective Differential Geometry of Submanifolds

M.A. AKIVIS

*Moscow Institute of Steel and Alloys
Department of Mathematics
Moscow, Russia*

V.V. GOLDBERG

*New Jersey Institute of Technology
Department of Mathematics
Newark, NJ, USA*



1993

NORTH-HOLLAND

AMSTERDAM • LONDON • NEW YORK • TOKYO

Table of Contents

Preface

v

CHAPTER 1

Preliminaries

1.1	Vector Spaces	1
1.2	Differentiable Manifolds	5
1.3	Projective Space	17
1.4	Some Algebraic Manifolds	26
Notes		31

CHAPTER 2

The Foundations of Projective Differential Geometry of Submanifolds

2.1	Submanifolds in a Projective Space and Their Tangent Subspaces	33
2.2	The Second Fundamental Form of a Submanifold	38
2.3	Osculating Subspaces and Fundamental Forms of Higher Orders of a Submanifold	42
2.4	Asymptotic and Conjugate Directions of Different Orders on a Submanifold	47
2.5	Some Particular Cases and Examples	53
2.6	Classification of Points of Submanifolds by Means of the Second Fundamental Form	61
Notes		70

CHAPTER 3

Submanifolds Carrying a Net of Conjugate Lines

3.1	Basic Equations and General Properties	73
3.2	The Holonomicity of the Conjugate Net Σ_2	76
3.3	Classification of Conjugate Nets Σ_2	81
3.4	Some Existence Theorems	85
3.5	The Laplace Transforms of Conjugate Nets and Their Generalizations	90
3.6	Conic m -Conjugate Systems	103
Notes		111

CHAPTER 4

Tangentially Degenerate Submanifolds

4.1	Basic Notions and Equations	113
4.2	Focal Images	117
4.3	Decomposition of Focal Images	120
4.4	The Holonomicity of the Focal Net	122
4.5	Some Other Classes of Tangentially Degenerate Submanifolds	126
4.6	Manifolds of Hypercones	130
4.7	Parabolic Submanifolds without Singularities in Euclidean and Non-Euclidean Spaces	132
Notes		141

CHAPTER 5

Submanifolds with Asymptotic and Conjugate Distributions

5.1	Distributions on Submanifolds of a Projective Space	143
5.2	Asymptotic Distributions on Submanifolds	145
5.3	Submanifolds with a Complete System of Asymptotic Distributions	148
5.4	Three-Dimensional Submanifolds Carrying a Net of Asymptotic Lines	151
5.5	Submanifolds with a Complete System of Conjugate Distributions	165
Notes		171

CHAPTER 6

Normalized Submanifolds in a Projective Space

6.1	The Problem of Normalization of a Submanifold in a Projective Space	173
6.2	The Affine Connection on a Normalized Submanifold	178
6.3	The Connection in the Normal Bundle	182
6.4	Submanifolds with a Flat Normal Connection	188
6.5	Intrinsic Normalization of Submanifolds	192
6.6	Normalization of Submanifolds Carrying a Conjugate Net of Lines	199
Notes		205

CHAPTER 7

Projective Differential Geometry of Hypersurfaces

7.1	Basic Equations of the Theory of Hypersurfaces	209
7.2	Osculating Hyperquadrics of a Hypersurface	216
7.3	Invariant Normalizations of a Hypersurface	222
7.4	The Rigidity Problem in a Projective Space	234
7.5	The Geometry of a Surface in Three-Dimensional Projective Space	245
7.6	The Geometry of Hyperbands	255
Notes		266

CHAPTER 8

Algebraization Problems in Projective Differential Geometry

8.1	The First Generalization of Reiss' Theorem	271
8.2	The Second Generalization of Reiss' Theorem	277
8.3	Degenerate Monge's Varieties	279
8.4	Submanifolds with Degenerate Bisecant Varieties	285
Notes		295

Bibliography	297
Symbols Frequently Used	333
Index	335