
Chaotic Transitions in Deterministic and Stochastic Dynamical Systems

*Applications of Melnikov Processes in
Engineering, Physics, and Neuroscience*

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