Abstract Convex Analysis

IVAN SINGER

Romanian Academy Bucharest, Romania



A Wiley-Interscience Publication

JOHN WILEY & SONS, INC.

New York . Chichester . Weinheim . Brisbane . Singapore . Toronto

Contents

Foreword			xi
Preface			xiii
Introduct	ion: Fro	on Convex Analysis to Abstract Convex Analysis	1
1.0	Abstra	ct Convexity of Sets	1
	0.1a	Inner Approaches	2
	0.16	Intersectional and Separational Approaches	4
	0.1c	Approaches via Convexity Systems and Hull Operators	6
0.2	Abstra	et Convexity of Functions	9
0.3	Abstract Convexity of Elements of Complete Lattices		
0.4	Abstra	ct Quasi-Convexity of Functions	14
0.5	Dualiti	es	14
0.6	Abstra	et Conjugations	16
0.7	Abstra	et Subdifferentials	19
0.8	Some Applications of Abstract Convex Analysis to Optimization		
	Theory	Appear to province - control messes establishmen in process and assessment	20
	0.8a	Applications to Abstract Lagrangian Duality	20
	0.86	Applications to Abstract Surrogate Duality	28
Chapter C	ne Ab	stract Convexity of Elements of a Complete Lattice	34
1.1	The Ma	ain (Supremal) Approach: M-Convexity of Elements of a	
	Compl	ete Lattice E , Where $\mathcal{M} \subseteq E$	34
1.2	Infimal and Supremal Generators and M-Convexity		
1.3			
1.4	Anothe	r Equivalent Approach: Convexity with Respect to a Hull	
	Operat	or	47
Chapter T	wo Ab	stract Convexity of Subsets of a Set	50
2.1	M-Cor	invexity of Subsets of a Set X, Where $\mathcal{M} \subseteq 2^{X}$	50
2.2	Some Particular Cases		56
	2.2a	Convex Subsets of a Linear Space X	56
	2.2b	Closed Convex Subsets of a Locally Convex Space X	58

vi Contents

	2.2c	Evenly Convex Subsets of a Locally Convex Space X		
	2.2d	Closed Affine Subsets of a Locally Convex Space X		
	2.2e	Evenly Confline Subsets of a Locally Convex Space X		
	2.26	Spherically Convex Subsets of a Metric Space X		
	2.2g	Closed Subsets of a Topological Space X		
	2.2h			
		Order Ideals and Order Convex Subsets of a Poset X		
	2.2i	Parametrizations of Families $\mathcal{M} \subseteq 2^x$, Where \mathcal{M} is a Set		
2,3	An Eq	An Equivalent Approach, via Separation by Functions:		
	W-Convexity of Subsets of a Set X, Where $W \subseteq \overline{\mathbb{R}}^X$			
2.4	A Particular Case: Closed Convex Sets Revisited			
2.5				
	to a Se	et of Functions $W \subseteq \overline{R}^X$		
2.6)-Convexity of Subsets of a Set X, Where W Is a Set and		
		$\times W \to \frac{1}{R}$ Is a Coupling Function		
Chapter T	Three .	Abstract Convexity of Functions on a Set		
3.1	W-Co	invexity of Functions on a Set X, Where $W \subseteq \overline{R}^X$		
3.2		Particular Cases		
	3.2a	$C(X^+ + R)$, Where X Is a Locally Convex Space		
	3.2b	$C(X^*)$, Where X is a Locally Convex Space		
	3.2c	The Case Where $X = \{0, 1\}^n$ and $W \subseteq (R^n)^* _X$		
	3.2d			
		The Case Where $X = \{0, 1\}^n$ and $W = (R^n)^* _X + R$		
	3.2e	α-Hölder Continuous Functions with Constant N,		
		Where $0 < \alpha \le 1$ and $0 < N < +\infty$		
	3.2f	Suprema of α -Hölder Continuous Functions, Where		
		$0 < \alpha \leqslant 1$		
	3.2g	The Case Where $\alpha > 1$		
3.3	(W, φ)-Convexity of Functions on a Set X, Where W Is a Set and			
	$\varphi:X$	$\times W \to \overline{R}$ Is a Coupling Function		
Chapter F	our A	bstract Quasi-Convexity of Functions on a Set		
4.1	M-O	iasi-Convexity of Functions on a Set X, Where $\mathcal{M} \subseteq 2^X$		
4.2		Particular Cases		
	4.20	Quasi-Convex Functions on a Linear Space X		
	4.2b	Lower Semicontinuous Quasi-Convex Functions on a		
	4120	Locally Convex Space X		
	4.2c	Evenly Quasi-Convex Functions on a Locally Convex		
	9100			
	121	Space X		
	4.20	Evenly Quasi-Coaffine Functions on a Locally Convex Space X		
	4.2c	Lower Semicontinuous Functions on a Topological		
	T. a.C.			
	126	Space X		
4.4	4.2f	Nondecreasing Functions on a Poset X		
4.3				
	Set X , Where $W \subseteq R^X$			

Contents vii

4.4	Relations Between W-Convexity and W-Quasi-Convexity of	1	
	Functions on a Set X , Where $W \subseteq \overline{\mathbb{R}}^X$		
4.5	Some Particular Cases		
	4.5a Lower Semicontinuous Quasi-Convex Functions Revisited		
	4.5b Evenly Quasi-Convex Functions Revisited		
	4.5c Evenly Quasi-Coaffine Functions Revisited		
4.6	(W, φ)-Quasi-Convexity of Functions on a Set X, Where W Is	a	
	Set and $\varphi: X \times W \to R$ is a Coupling Function		
4.7	Other Equivalent Approaches: Quasi-Convexity of Functions of	n	
	a Set X, with Respect to Convexity Systems $\mathcal{B} \subseteq 2^X$ and Hul		
	Operators $u: 2^X \rightarrow 2^X$		
4.8	Some Characterizations of Quasi-Convex Hull Operators		
	among Hull Operators on \overline{R}^{χ}		
Chapter F	ive Dualities Between Complete Lattices		
5.1	Dualities and Infimal Generators		
5.2	Duals of Dualities		
5.3	Relations Between Dualities and M-Convex Hulls		
5.4	Partial Order and Lattice Operations for Dualities		
Chapter S	ix Dualities Between Families of Subsets		
6.1	Dualities $\Delta: 2^X \to 2^W$, Where X and W Are Two Sets		
6.2	Some Particular Cases		
	6.2a Some Minkowski-Type Dualities		
	6.2b Some Dualities Obtained from the Minkowski-Typ	e	
	Dualities Δ_M , by Parametrizing the Family M		
6.3			
	Subsets Ω of $X \times W$ and Coupling Functions $\varphi : X \times W \to \overline{R}$		
6.4	THE PURPOSE OF THE PROPERTY OF THE PURPOSE OF THE P		
	6.4a Representations with the Aid of Subsets Ω of $X \times W$		
	6.4b Representations with the Aid of Coupling Functions		
	$\varphi: X \times W \to \overline{R}$		
	A THE COURT OF THE		
Chapter S	even Dualities Between Sets of Functions		
7.1	Dualities $\Delta : \overline{R}^X \to \overline{R}^W$, Where X and W Are Two Sets		
7.2	Representations of Dualities $\Delta: A^X \to F$, Where X Is a Sc	1	
1000	and $(A, \leq) \subseteq (\overline{R}, \leq)$ and F Are Complete Lattices	51/1	
7.3	Dualities $\Delta : A^X \to B^W$, Where X Is a Set and (A, \leq) .		
8.7	$(B, \leqslant) \subseteq (\overline{R}, \leqslant)$ Are Complete Lattices		
7.4	Some Particular Cases		
	7.4a The Case Where $A = \{0, +\infty\}$		
	7.4b The Case Where $A = \{0, +\infty\}$		
7.5	Strict Dualities $\Delta : A^X \to B^W$		
7.6	Dualitylike Mappings $\Delta : A^X \to B^W$		
7,43	Louding that Proppings 11 . At D		

viii Contents

Chapter E	ight Conjugations				
8.1	Conjugations $c: \overline{R}^X \to \overline{R}^W$, Where X and W Are Two Sets				
8.2					
	Coupling Functions $\varphi: X \times W \to \overline{R}$				
8.3					
8.4	4-00-00-00 Bit 18-00-00-00 March 19-40-00-00-00-00-00-00-00-00-00-00-00-00-				
05.4	8.4a The Case Where $X = \{0, 1\}^n$, $W = (R^n)^n _X$ and				
	$\varphi = \varphi_{\text{tot}}$				
	8.4b The Case Where X Is a Metric Space, $W = X$,				
	and $\varphi = \varphi_{n,N}$				
	8.4c The Case Where X Is a Metric Space, $W = X \times$				
	$(R_+ \setminus \{0\})$, and $\varphi = \varphi_0$				
0.6					
8.5	Conjugations of Type Lau				
0, 1	Some Particular Cases				
	8.7a Conjugations of Type Lau Associated to a Family M of				
	Subsets of a Set X				
	8.7b Quasi-Conjugation				
	8.7c Semiconjugation				
	8.7d Pseudoconjugation				
	8.7e Some Extensions of the Preceding Conjugations				
8.8					
	$\Delta: 2^X \to 2^W$, Where X and W Are Two Sets				
8.9	Some Particular Cases				
	8.9a The Conjugation of Type Lau Associated to a				
	Minkowski-Type Duality				
	8.9b Conjugations of Type Lau Associated to Parametrized				
	Minkowski-Type Dualities				
8.10	The Conjugate of Type Lau of $\max\{f, -h\}$. Where $f, -h \in \overline{R}^X$				
8.11	Conjugate Functions and Level Sets				
Chapter !	Vine V-Dualities and ⊥-Dualities				
9.1	The Binary Operations ⊥ and ⊤				
9.2	∨-Dualities				
9.3	L-Dualities •				
9.4	The Duals of ∨-Dualities				
9.5	The Duals of ⊥-Dualities				
9.6	Characterizations of Conjugations of Type Lau with the Aid of				
	∨-Dualities and ⊥-Dualities				
Chapter 7	len Abstract Subdifferentials				
10.1	Subdifferentials with Respect to a Duality $\Delta : \overline{R}^X \to \overline{R}^W$,				
2750000	Where X and W Are Two Sets				
10.2	20대 [[10대]				
	Where X and W Are Two Sets				
	THE PARTY OF THE P				

Centents

10.3	Some Particular Cases		36	
	10.3a	The Case Where $X = \{0, 1\}^s$, $W = (R^s)^s _X$, and		
		$\varphi = \varphi_{uv}$	36	
	10.3b	The Case Where X Is a Metric Space, $W = X$, and		
		$\psi = \varphi_{w,N}$	36	
	10.3c	The Case Where X Is a Metric Space, $W = X \times$		
		$(R_+ \setminus \{0\})$, and $\varphi = \varphi_{\sigma}$	36	
10.4	The Subdifferential of $f + h$ at x_0 . Where $f, h \in \mathbb{R}^X$ and			
	$x_0 \in \lambda$		37	
10.5	Subdiff	Subdifferentials with Respect to Conjugations of Type Lau		
10.6	Some Particular Cases		37	
	10.6a	$L(\Delta)$ -Subdifferentials, for Minkowski-Type and		
		Parametrized Minkowski-Type Set-Dualities A	37	
	10.6b	Subdifferentials with Respect to Quasi-Conjugations;		
		Quasi-Subdifferentials	37	
	10.6c	Subdifferentials with Respect to Semiconjugations;		
		Semisubdifferentials	37	
	10.6d	Subdifferentials with Respect to Pseudoconjugations;		
		Pseudosubdifferentials	38	
10.7	Subdif	ferentials with Respect to ∨-Dualities and ⊥-Dualities	38	
Notes and	l Remar	ks	38	
Reference	es		46	
Notation .	Index		47	
Author In	idex		48	
Subject I	ader		48	