DIFFERENTIAL GEOMETRY

J. J. STOKER

Wiley Classics Edition Published in 1989



WILEY-INTERSCIENCE

a Division of John Wiley & Sons New York · London · Svdnev · Toronto

CONTENTS

Chapte	r I Operations with Vo	ectors								
1.	The vector notation	104	240	21415	:40	400		4.0	0.04	1
2.	Addition of vectors			4	4	4.3	4			2
3.	Multiplication by scala	rs.				+3				3
4.	Multiplication by scala Representation of a vec	tor by	y mean	ns of li	nearly	indep	pende	nt vect	ors	3
5.	Scalar product .		4						4	3
6.	Scalar product	1	1		+			-		5
7.	Scalar triple product	100	12		2.3	20		-	1	6
8.	Invariance under ortho	gonal	l trans	sforma	tions		100	200	107	7
9.	Vector calculus .		+			*		+		9
Chapte	r II Plane Curves									
	Introduction						0	+	-	12
2.	Regular curves .									12
3.	Change of parameters									14
4.	Regular curves . Change of parameters Invariance under change	ges of	para	meter			4			16
5.	Tangent lines and tang	ent w	ectors	of a c	curve	40			-	16
6.	Orientation of a curve		1.5			200				18
7.	Length of a curve . Arc length as an invar		4			-	-			19
8.	Arc length as an invar	ant	- 6	13	4			4.3		20
9.	Curvature of plane cur	rves			1.4	+	-	+		21
10.	The normal vector and	the s	sign o	fx.	170	7.0	0.7	200		23
11.	Formulas for * _		4				+			26
	Existence of a plane cu									27
13.	Frenet equations for p	lane c	urves			4				28
	Evolute and involute of									29
	Envelopes of families									31
16.	The Jordan theorem a	s a p	robles	m in d	ifferen	tial g	eome	try in	the	
	large									34
17.	Additional properties								000	41
	The total curvature of									45
19.	Simple closed curves	with	< ≠ 0	as bo	ounda	ries o	f con	vex pe	oint	
	sets									46
20.	Four vertex theorem		4	6						48

xviii CONTENTS

Chapter III S	pace Curves
---------------	-------------

2.	Length of a curve .									
3							1.5			5
	Curvature of space cur	ves			4		14		4	54
4.	Length of a curve . Curvature of space cur Principal normal and o	scular	ting p	lane						55
5.	Binormal vector .				,		0.00	0.70		57
6.	Torsion + of a space cu	irve					+			57
7.	Binormal vector . Torsion + of a space cu The Frenet equations for	or spe	ice ci	irves						58
8.	Rigid body motions an	d the	rotat	ion ve	ctor		204	2940	* 3	58
9.	Rigid body motions an The Darboux vector Formulas for * and *			4			4		- 7	62
10.	Formulas for * and *		4					-	+	63
11.	The sign of r		20.06		10.0	0.04	10.0		+**	63
12	Canonical representation	on of	a cur	ve .	+					64
13.	Canonical representation Existence and uniquene	ss of a	space	e curv	e for g	iven	k(s), †	(3).		65
14.	What about $\kappa = 0$?			4					+1	67
15.	Another way to define	space	curv	es .						68
16.	What about $\kappa = 0$? Another way to define Some special curves							*	٠	70
. c. 5.c.	r IV The Basic Elemen				2,000					
1.	Regular surfaces in Euc	clidea	n spa	ce .	+	. 4	+	+		74
2.	Change of parameters Curvilinear coordinate				3.7%	-	30.0	2	70	75
3.	Curvilinear coordinate	curve	s on a	a surfa	ce		+	+	+	76
4.	Tangent plane and non	mal ve	ector	· · · · ·				4		77
5.	Length of curves and fi Invariance of the first f	rst fu	ndam	ental f	orm	17		7.1	1.1	78
6.	Length of curves and fi Invariance of the first f Angle measurement on	undar	nenta	d form		+	*	+	+	78
7.	Angle measurement on	surfa	ces	+		+	+			80
8.	Area of a surface . A few examples . Second fundamental for	135		253	500	13	*	333	50	82
9.	A few examples .	*	(+)			+	+			83
10.	Second fundamental for	rm of	a sur	face						85
11.	Osculating paraboloid	98 g	0.00		500	328	1.5	250		86
12.	Curvature of curves on	a sur	face	4			+	+		88
13.	Principal directions and	princ	cipal	curvat	ures	12				91
14.	Mean curvature H and	Gaus	sian o	curvati	ire K	106	1.40	*	100	92
15.	Another definition of the	ne Ga	ussiai	n curva	ture .	K.				93
16.	Osculating paraboloid Curvature of curves on Principal directions and Mean curvature H and Another definition of the Lines of curvature.	4								95
11.	i niru tungamentai torn	n .				. +	+			30
18. 6	Characterization of the	spher	e as a	locus	of um	bilica	d poin	ts.		99
19.	Asymptotic lines . Torsion of asymptotic l									100
20.	Torsion of asymptotic l	ines	4				4			100
21. 1	Introduction of special	paran	neter	curves						101
22.	Introduction of special Asymptotic lines and lin	nes of	curv	ature a	s par	amete	er curv	es	-	103
							*****			103
23. 1	Embedding a given arc	in a s	ysten	i or pa	ramet	er cu	LVCS			100

CONTENTS XIX

	v Some Special Surfaces							
1.	Surfaces of revolution			S				109
2.	Developable surfaces in the small n	nade u	p of p	arabol	lic poi	nts	000	114
	Edge of regression of a developable							
4.	Why the name developable? .					,		120
5.	Developable surfaces in the large							121
6.	Developable surfaces in the large Developables as envelopes of plans	cs.						129
Chapter	VI The Partial Differential Equa	itions o	of Surf	face T	heory			
1.	Introduction	100					+	133
2.	The Gauss equations	4						134
3.	The Christoffel symbols evaluated				1	4	+	
4.	The Weingarten equations .							136
5.	Some observations about the parti	ial diffe	erentia	al equa	tions		+	136
6.	Uniqueness of a surface for given	gik and	1 Lik		10		12	138
7.	The theorema egregium of Gauss							139
8.	How Gauss may have hit upon his	s theor	rem				-	141
9.	Compatibility conditions in general Codazzi-Mainardi equations	al .					+	143
10.	Codazzi-Mainardi equations .							144
	The Gauss theorema egregium aga						940	
12.	Existence of a surface with given a	gu and	L_{tk}					146
13.	An application of the general theor	ry to a	proble	em in t	the lar	ge		148
Chapte	r VII Inner Differential Geometry Point of View	in the	e Smal	ll from	the l	extrin	sic	
	Introduction. Motivations for th					34		151
2.	Approximate local parallelism of	vectors	s in a	surfac	c .	4		155
3.	Parallel transport of vectors along	curves	in the	sense	of Le	vi-Ci	vita	157
4.	Properties of parallel fields of vect	tors ale	ong cu	irves				160
5.	Parallel transport is independent of	of the p	ath o	nly for	surfa	ices h	av-	
	ing K = 0							162
6.	The curvature of curves in a surface	ce: the	geode	tic cur	vatur	е.		163
7.	First definition of geodetic lines:	lines w	ith Ka	= 0				165
	Geodetic lines as candidates for sl							167
	Straight lines as shortest arcs in the					10.01	0.4	168
10.	A general necessary condition for	a sho	rtest a	rc.				171
11.	Geodesics in the small and geodes	tic coo	rdinat	e syste	ems	4.00		174
12.	Geodesics as shortest arcs in the s	small						178
13	Further developments relating to a	reodeti	ic coor	rdinate	syste	ms		179
14.	Surfaces of constant Gaussian cur	rvature						183
13.	Paranet neits from a new point o	T TIME TO						
16.	Models provided by differentia	d geor	metry	for	non-E	uclid	ean	
200	geometries		1					185
17	Parallel transport of a vector arou	nd a si	mple o	closed	curve			191

XX CONTENTS

Derivation of the Gauss-Bonnet formula .	2.4	25	.40	+0	195
19. Consequences of the Gauss-Bonnet formula	3.0			+	196
20. Tchebychef nets	4	٠		٠	198
Chapter VIII Differential Geometry in the Large					
1. Introduction. Definition of n-dimensional m	anifol	ds.		40	203
2. Definition of a Riemannian manifold .					206
3. Facts from topology relating to two-dimensio	nal m	anifolo	is.		211
4. Surfaces in three-dimensional space			+	+	217
5. Abstract surfaces as metric spaces	1				218
6. Complete surfaces and the existence of shorte	st arc	S .			220
7. Angle comparison theorems for geodetic triar	gles		4		220
8. Geodetically convex domains					23
The Gauss-Bonnet formula applied to closed	surfac	ces	30	- 3	237
Vector fields on surfaces and their singularities	· .		+		239
11. Poincaré's theorem on the sum of the indices			ırface	5 .	244
Conjugate points. Jacobi's conditions for sh					24
The theorem of Bonnet-Hopf-Rinow .					
Synge's theorem in two dimensions	0.6	-		+.	25
Covering surfaces of complete surfaces having					259
16. Hilbert's theorem on surfaces in E^3 with $K =$					263
The form of complete surfaces of positive curv	ature	in thre	e-dim		
sional space	12		4		27
Chapter IX Intrinsic Differential Geometry of Manif	olds.	Relat	ivity		
1. Introduction					283
			, in	3.3	
Part I. Tensor Calculus in Affine and Eucl	lidean	Space	5		
Affine geometry in curvilinear coordinates.					28
3. Tensor calculus in Euclidean spaces					28
4. Tensor calculus in mechanics and physics .					29
Part II. Tensor Calculus and Differential General Manifolds	Geom	etry in	i.		
					20
5. Tensors in a Riemannian space	2	1.7	(*)		29
6. Basic concepts of Riemannian geometry .		alidaa			-
 Parallel displacement. Necessary condition : Normal coordinates. Curvature in Riemann 					
Normal coordinates. Curvature in Riemann Geodetic lines as shortest connections in the				*	
Geodetic lines as shortest connections in the Geodetic lines as shortest connections in the				+	31
		135		*	31
Part III. Theory of Relativity					
11. Special theory of relativity		0.7			31
12. Relativistic dynamics					32
 The general theory of relativity 					32

XXI

5. Minimal surfaces 6. Uniqueness theorems for closed convex surfaces 7. Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 7. Introduction 7. Geometry in an affine space 7. Tensor algebra in centered affine spaces 7. Effect of a change of basis 7. Definition of tensors 7. Tensor algebra in Euclidean spaces 8. Tensor algebra in Euclidean spaces 9. Appendix B Differential Equations 9. Theorems on ordinary differential equations 9. Overdetermined systems of linear partial differential equations 9. Overdetermined systems of linear partial differential equations		1	The Wedge Forms, with	App	dication	s to S	ırfac	e Theo	ry				
3. Scalar and vector products of vector forms on surfaces and their exterior derivatives. 4. Some formulas for closed surfaces. Characterizations of the sphere. 5. Minimal surfaces. 6. Uniqueness theorems for closed convex surfaces. Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces. 1. Introduction. 2. Geometry in an affine space. 3. Tensor algebra in centered affine spaces. 4. Effect of a change of basis. 5. Definition of tensors. 6. Tensor algebra in Euclidean spaces. Appendix B Differential Equations 1. Theorems on ordinary differential equations. 2. Overdetermined systems of linear partial differential equations.	1.	Defin	itions .	100.00	2500					2			1
exterior derivatives. 4. Some formulas for closed surfaces. Characterizations of the sphere 5. Minimal surfaces 6. Uniqueness theorems for closed convex surfaces Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 1. Introduction 2. Geometry in an affine space 3. Tensor algebra in centered affine spaces 4. Effect of a change of basis 5. Definition of tensors 6. Tensor algebra in Euclidean spaces Appendix B Differential Equations 1. Theorems on ordinary differential equations 2. Overdetermined systems of linear partial differential equations	2.	Vecto	r differenti	al fo	rms and	i surfa	ce th	neory	4				
4. Some formulas for closed surfaces. Characterizations of the sphere 5. Minimal surfaces 6. Uniqueness theorems for closed convex surfaces Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 1. Introduction 2. Geometry in an affine space 3. Tensor algebra in centered affine spaces 4. Effect of a change of basis 5. Definition of tensors 6. Tensor algebra in Euclidean spaces Appendix B Differential Equations 1. Theorems on ordinary differential equations 2. Overdetermined systems of linear partial differential equations	3.	Scalar	and vecto	or pro	oducts o	of vect	tor f	orms o	on sur	faces a	nd t	their	
4. Some formulas for closed surfaces. Characterizations of the sphere. 5. Minimal surfaces. 6. Uniqueness theorems for closed convex surfaces. Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 1. Introduction. 2. Geometry in an affine space. 3. Tensor algebra in centered affine spaces. 4. Effect of a change of basis. 5. Definition of tensors. 6. Tensor algebra in Euclidean spaces. Appendix B Differential Equations 1. Theorems on ordinary differential equations. 2. Overdetermined systems of linear partial differential equations.						+	40	+	*00	100	600		3
5. Minimal surfaces 6. Uniqueness theorems for closed convex surfaces 7. Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 1. Introduction 2. Geometry in an affine space 3. Tensor algebra in centered affine spaces 4. Effect of a change of basis 5. Definition of tensors 6. Tensor algebra in Euclidean spaces 1. Theorems on ordinary differential equations 1. Theorems on ordinary differential equations 2. Overdetermined systems of linear partial differential equations	4.	Some	formulas	for	closed	surfa	ces.	Char	acteri	zations	of	the	
5. Minimal surfaces 6. Uniqueness theorems for closed convex surfaces Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 1. Introduction 2. Geometry in an affine space 3. Tensor algebra in centered affine spaces 4. Effect of a change of basis 5. Definition of tensors 6. Tensor algebra in Euclidean spaces Appendix B Differential Equations 1. Theorems on ordinary differential equations 2. Overdetermined systems of linear partial differential equations		spher	e	0.46	240		750	950	+:	2.0	100		
Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 1. Introduction		Minir	mal surface	s.	4		+		+				3
Appendix A Tensor Algebra in Affine, Euclidean, and Minkowski Spaces 1. Introduction	6.	Uniqu	ueness theo	rems	for clo	sed co	nve	surfa	ces				
Tensor algebra in Euclidean spaces	3. 4.	Effec	or algebra i t of a chan	in cer ge of	ntered a basis	iffine s	pace	s .					
Appendix B Differential Equations 1. Theorems on ordinary differential equations 2. Overdetermined systems of linear partial differential equations 3												*	
Theorems on ordinary differential equations Overdetermined systems of linear partial differential equations	6.	Tense	or algebra i	in Eu	iclidean	space	5 .	+	+				
Overdetermined systems of linear partial differential equations .	ppen	dix B	Differentia	al Eq	uations								
Overdetermined systems of linear partial differential equations .	1.	Theo	rems on or	dina	rv differ	rential	equa	ations					
Bibliography	2.	Over	determined	syste	ems of l	linear	parti	al diffe	erentia	d equat	ion	s .	
	Biblio	graph	у			60	25		٠		٠		