Stefan Teufel

Adiabatic Perturbation Theory in Quantum Dynamics

Springer

This one
QNEU-B1A-Y24T

Table of Contents

1 Introduction 1
- 1.1 The time-adiabatic theorem of quantum mechanics 6
- 1.2 Space-adiabatic decoupling: examples from physics 15
  - 1.2.1 Molecular dynamics 15
  - 1.2.2 The Dirac equation with slowly varying potentials 21
- 1.3 Outline of contents and some left out topics 27

2 First order adiabatic theory 33
- 2.1 The classical time-adiabatic result 33
- 2.2 Perturbations of fibered Hamiltonians 39
- 2.3 Time-dependent Born-Oppenheimer theory: Part I 44
  - 2.3.1 A global result 46
  - 2.3.2 Local results and effective dynamics 50
  - 2.3.3 The semiclassical limit: first remarks 57
  - 2.3.4 Born-Oppenheimer approximation in a magnetic field and Berry's connection 61
- 2.4 Constrained quantum motion 62
  - 2.4.1 The classical problem 62
  - 2.4.2 A quantum mechanical result 65
  - 2.4.3 Comparison 67

3 Space-adiabatic perturbation theory 71
- 3.1 Almost invariant subspaces 75
- 3.2 Mapping to the reference space 83
- 3.3 Effective dynamics 89
  - 3.3.1 Expanding the effective Hamiltonian 92
- 3.4 Semiclassical limit for effective Hamiltonians 95
  - 3.4.1 Semiclassical analysis for matrix-valued symbols 96
  - 3.4.2 Geometrical interpretation: the generalized Berry connection 101
  - 3.4.3 Semiclassical observables and an Egorov theorem 102

4 Applications and extensions 105
- 4.1 The Dirac equation with slowly varying potentials 105
  - 4.1.1 Decoupling electrons and positrons 106

4.1.2 Semiclassical limit for electrons: the T-BMT equation . 111
4.1.3 Back-reaction of spin onto the translational motion ... 115
4.2 Time-dependent Born-Oppenheimer theory: Part II 124
4.3 The time-adiabatic theorem revisited 127
4.4 How good is the adiabatic approximation? 131
4.5 The B.-O. approximation near a conical eigenvalue crossing .. 136

5 Quantum dynamics in periodic media 141
5.1 The periodic Hamiltonian 145
5.2 Adiabatic perturbation theory for Bloch bands 151
5.2.1 The almost invariant subspace 155
5.2.2 The intertwining unitaries 159
5.2.3 The effective Hamiltonian 161
5.3 Semiclassical dynamics for Bloch electrons 163

6 Adiabatic decoupling without spectral gap 173
6.1 Time-adiabatic theory without gap condition 174
6.2 Space-adiabatic theory without gap condition 178
6.3 Effective N-body dynamics in the massless Nelson model 185
6.3.1 Formulation of the problem 185
6.3.2 Mathematical results 193

A Pseudodifferential operators 203
A.1 Weyl quantization and symbol classes 203
A.2 Composition of symbols: the Weyl-Moyal product 208

B Operator-valued Weyl calculus for τ-equivariant symbols . 215

C Related approaches 221
C.1 Locally isospectral effective Hamiltonians 221
C.2 Simultaneous adiabatic and semiclassical limit 223
C.3 The work of Blount and of Littlejohn et al. 224

List of symbols 225

References 227

Index 235