

Infinite-Dimensional Topology

Prerequisites and Introduction

J. van Mill

*Vrije Universiteit
Amsterdam, The Netherlands*



NORTH-HOLLAND
AMSTERDAM · NEW YORK · OXFORD · TOKYO

Table of Contents

Chapter 1: Extension Theorems

§1.1. <i>Topological Spaces</i>	1
§1.2. <i>Linear Spaces</i>	5
§1.3. <i>Function Spaces</i>	18
§1.4. <i>The Michael Selection Theorem and Applications</i>	26
§1.5. <i>AR's and ANR's</i>	44
§1.6. <i>The Borsuk Homotopy Extension Theorem</i>	50
Notes	56

Chapter 2: Elementary Plane Topology

§2.1. <i>The Brouwer Fixed-Point Theorem and Applications</i>	59
§2.2. <i>The Borsuk-Ulam Theorem</i>	68
§2.3. <i>The Poincaré Theorem</i>	72
§2.4. <i>The Jordan Curve Theorem</i>	77
Notes	85

Chapter 3: Elementary Combinatorial Techniques

§3.1. <i>Affine Notions</i>	87
§3.2. <i>Simplexes</i>	91
§3.3. <i>Triangulation</i>	96
§3.4. <i>Simplexes in $\mathbb{R}^n$</i>	101
§3.5. <i>The Brouwer Fixed-Point Theorem</i>	105
§3.6. <i>Topologizing a Simplicial Complex</i>	111
Notes	123

Chapter 4: Elementary Dimension Theory

§4.1. <i>The Covering Dimension</i>	125
§4.2. <i>Zero-dimensional Spaces</i>	131
§4.3. <i>Translation into Open Covers</i>	140
§4.4. <i>The Imbedding Theorem</i>	150
§4.5. <i>The inductive Dimension Functions ind and Ind</i>	156
§4.6. <i>Mappings into Spheres</i>	164
§4.7. <i>Totally Disconnected Spaces</i>	171
§4.8. <i>Various kinds of Infinite-Dimensionality</i>	181
Notes	189

Chapter 5: Elementary ANR theory

§5.1. <i>Some Properties of ANR's</i>	191
§5.2. <i>A Characterization of ANR's and AR's</i>	199
§5.3. <i>Hyperspaces and the AR-Property</i>	213
§5.4. <i>Open Subspaces of ANR's</i>	226

§5.5. Characterization of Finite-Dimensional ANR's and AR's	232
§5.6. Adjunction Spaces of Compact $A(N)R$'s	238
Notes	247

Chapter 6: An Introduction to Infinite-Dimensional Topology

§6.1. Constructing New Homeomorphisms from Old	249
§6.2. Z-sets	262
§6.3. The Estimated Homeomorphism Extension Theorem for compacta in s	265
§6.4. The Estimated Homeomorphism Extension Theorem	274
§6.5. Absorbers	280
§6.6. Hilbert Space is Homeomorphic to the Countable Infinite Product of Lines	288
§6.7. Inverse Limits	299
§6.8. Hilbert Cube Factors	304
Notes	310

Chapter 7: Cell-like Maps and Q-Manifolds

§7.1. Cell-like Maps and Fine Homotopy Equivalences	313
§7.2. Z-sets in ANR's	320
§7.3. The Disjoint-Cells Property	330
§7.4. Z-Sets in Q-manifolds	337
§7.5. Toruńczyk's Approximation Theorem and Applications	345
§7.6. Cell-like Maps I	358
§7.7. Cell-like Maps II	364
§7.8. The Characterization Theorem	370
Notes	372

Chapter 8: Applications

§8.1. Infinite Products	375
§8.2. Keller's Theorem	376
§8.3. Cone Characterization of the Hilbert Cube	380
§8.4. The Curtis-Schori-West Hyperspace Theorem	381
Notes	387

What Next ?	388
Bibliography	389
Subject Index	395