

Convexity

ROGER WEBSTER

*Department of Pure Mathematics,
Sheffield University*

Oxford New York Tokyo
OXFORD UNIVERSITY PRESS
1994

Contents

| | |
|---|-----|
| Introduction | xv |
| 1 The Euclidean space \mathbb{R}^n | 1 |
| 1.1 The Euclidean space \mathbb{R}^n | 1 |
| 1.2 Flats | 4 |
| 1.3 Dimension | 10 |
| 1.4 Hyperplanes | 17 |
| 1.5 Affine transformations | 22 |
| 1.6 Length, distance, and angle | 25 |
| 1.7 Open sets and closed sets | 32 |
| 1.8 Convergence and compactness | 38 |
| 1.9 Continuity | 43 |
| 2 Convex sets | 49 |
| 2.1 Basic properties of convex sets | 49 |
| 2.2 The convex hull | 54 |
| 2.3 Interiors and closures | 61 |
| 2.4 Separation and support | 65 |
| 2.5 Unbounded convex sets | 73 |
| 2.6 Facial structure | 79 |
| 2.7 The Blaschke selection principle | 90 |
| 2.8 Duality | 99 |
| 3 Convex polytopes | 105 |
| 3.1 Polytopes | 105 |
| 3.2 Polyhedral sets | 110 |
| 3.3 Pyramids, bipyramids, and prisms | 115 |
| 3.4 Cyclic polytopes | 120 |
| 3.5 Euler's relation | 127 |
| 3.6 Gale transforms | 135 |
| 4 Linear programming | 145 |
| 4.1 The finite basis theorem | 145 |
| 4.2 Linear inequalities | 149 |
| 4.3 Linear programming | 156 |
| 4.4 Basic solutions of linear equations | 163 |

| | | |
|-----|--|-----|
| 4.5 | The simplex algorithm | 168 |
| 4.6 | Game theory | 181 |
| 5 | Convex functions | 193 |
| 5.1 | Convex functions on the real line | 193 |
| 5.2 | Classical inequalities | 200 |
| 5.3 | The gamma and beta functions | 206 |
| 5.4 | Convex functions on \mathbb{R}^n | 217 |
| 5.5 | Continuity and differentiability of convex functions | 224 |
| 5.6 | Support functions | 231 |
| 5.7 | The convex programming problem | 239 |
| 5.8 | Matrix inequalities | 244 |
| 6 | Mixed volumes and extremum problems | 254 |
| 6.1 | Elementary sets | 254 |
| 6.2 | Volume | 262 |
| 6.3 | The determination of volume | 273 |
| 6.4 | Mixed volumes and surface area | 281 |
| 6.5 | The Brunn-Minkowski theorem | 297 |
| 6.6 | Steiner symmetrization | 306 |
| 7 | Selected topics | 316 |
| 7.1 | Helly's theorem | 316 |
| 7.2 | Join spaces | 330 |
| 7.3 | Convexity-preserving mappings | 343 |
| 7.4 | The geometry of numbers | 350 |
| 7.5 | Brouwer's fixed-point theorem | 356 |
| 7.6 | Convex bodies of constant width | 367 |
| 8 | Solutions to exercises | 377 |
| 9 | Notes on further reading | 425 |
| | Bibliography | 431 |
| | Index of symbols and notation | 437 |
| | General index | 440 |