

TOPOLOGICAL FIELDS

WITOLD WIĘŚLAW

*Wrocław University
Wrocław, Poland*

MARCEL DEKKER, INC.

S. U. N.

New York and Basel

Contents

PREFACE	iii
1. DEFINITIONS AND EXAMPLES	1
1. Bases of Zero Neighborhoods	1
2. Examples of Topological Fields	7
3. The Lattice $L_T(K)$ of Ring Topologies for a Field K	16
4. Existence of Field Topologies on Infinite Fields	20
5. Locally Bounded and Locally Unbounded Topologies	25
6. Topological Fields and Geometry	33
Notes	35
2. FUNDAMENTAL PROPERTIES OF ABSOLUTE-VALUED FIELDS	37
1. Artin-Whaples' Approximation Theorem, Hensel's Lemma	37
2. Description of Absolute Values of Some Fields	42
3. Topological Characterizations of Absolute-Valued Fields	50
4. Algebraic Characterizations of Absolute Values	56
5. Extensions of Absolute-Valued Fields	59
Notes	65
3. NORMED FIELDS AND THEIR PROPERTIES	67
1. Examples and Properties of Normed Fields	67
2. The Gelfand-Mazur Theorem	70
3. Topological Characterizations of Normed Fields	74
4. Regular and Nonregular Norms on Fields	83
5. Norms on the Quotient Fields of Dedekind Domains	91
6. Examples of Fields Complete with Respect to Norms Inequivalent to Absolute Values	107
Notes	110
4. KRULL VALUATIONS	116
1. Fundamental Properties of Valuations	116
2. Independence Theorems	122

3. Topological Characterization of Fields with Valuations	123
4. Extensions of Valuations	127
Notes	128
5. TOPOLOGIES OF TYPE V	130
1. Fundamental Properties	130
2. Characterization of Topologies of Type V	134
3. Minimal Locally Bounded Ring Topologies on Fields	138
4. Strictly Minimal Topological Fields	141
Notes	145
6. LOCALLY COMPACT TOPOLOGICAL FIELDS	148
1. Examples of Locally Compact Fields	148
2. Characterizations of Locally Compact Fields	151
Notes	158
7. INDEPENDENT TOPOLOGIES ON FIELDS	159
1. Examples of Independent Topologies	159
2. Some Results on Independent Field Topologies	162
3. Topologies Bounded by Suprema of V-Topologies	167
Notes	171
8. FIELDS COMPLETE IN TWO INDEPENDENT TOPOLOGIES	172
1. The Absolute Value Case	172
2. The Valuation Case	180
Notes	182
9. LOCALLY BOUNDED RING TOPOLOGIES ON FIELDS	183
1. Several Generalizations of Valuations	183
2. Nakano's Valuations	186
3. Locally Bounded Ring Topologies on Some Classes of Fields	195
4. Topological Characterizations of \mathbb{R} and \mathbb{C}	208
Note	211
10. CONNECTED FIELDS OF ARBITRARY CHARACTERISTIC	213
Notes	220
11. RING TOPOLOGIES ON THE RATIONAL NUMBER FIELD	222
12. LINEAR FIELD AND RING TOPOLOGIES ON THE QUOTIENT FIELD OF ARITHMETICAL RINGS	227
13. LOCALLY UNBOUNDED TOPOLOGIES ON FIELDS	235
14. AN EXTENSION PROBLEM FOR TOPOLOGICAL FIELDS	241
15. OPEN PROBLEMS	251

Contents	vii
APPENDIX: LINEARLY ORDERED GROUPS	258
NOTATION	262
BIBLIOGRAPHY	265
AUTHOR INDEX	301
SUBJECT INDEX	307