

Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 89

Billiards

A Genetic Introduction
to the Dynamics
of Systems with Impacts

Valerii V. Kozlov

Dmitrii V. Treshchëv



American Mathematical Society
Providence, Rhode Island

Contents

<i>Preface</i>	xii
INTRODUCTION. Elements of Impact Theory	1
<i>Problems</i>	27
CHAPTER I. The Generic Method in the Dynamics of Systems with One-sided Constraints	31
§1. One-sided and two-sided constraints	31
§2. Inelastic impact and a Kelvin-Voigt medium	36
§3. The limit theorem	39
§4. Averaging in systems with impacts	46
§5. Some generalizations	49
<i>Problems</i>	51
CHAPTER II. Periodic Trajectories of the Birkhoff Billiard	53
§1. Birkhoff's Theorem	53
§2. Proof of Birkhoff's Theorem	55
§3. Some generalizations	62
§4. The Hesse and Poincaré matrices	63
§5. Multipliers and the Hessian	67
§6. Conditions for stability of periodic trajectories	70
<i>Problems</i>	78
CHAPTER III. The Hill Equation	81
§1. Variational equations	81
§2. The Lyapunov constant	87
§3. Stability conditions	89
§4. Hill's method	92
<i>Problems</i>	95
CHAPTER IV. Integrable Problems	97
§1. The elliptic billiard	97
§2. Integrable billiards on ellipsoids	102
§3. The parabolic billiard	107
§4. The harmonic oscillator and the elliptic billiard	109

§5. Billiards in Weyl Chambers	111
Problems	116
CHAPTER V. Nonintegrable Billiards	117
§1. Nonintegrability of the typical Birkhoff billiard	117
§2. Polynomial integrals and the topology of billiards	129
§3. Integrable billiards on surfaces of constant curvature	133
§4. Ergodic properties of billiards	141
Problems	145
Appendix I. Systems with Elastic Reflections and KAM Theory	147
Appendix II. On the Connection of Dynamic and Geometric Properties of Periodic Trajectories	155
Bibliography	163
Subject Index	169