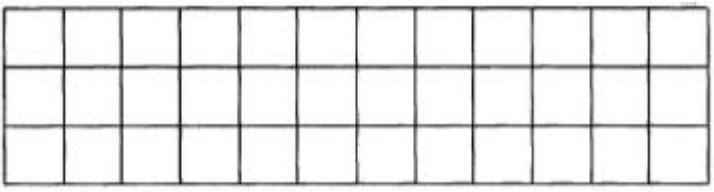

GRAPHS

AN INTRODUCTORY APPROACH

A First Course in Discrete Mathematics

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Based on The Open University course *Graphs, Networks and Design*

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