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AFTERNOTES goes to GRADUATE SCHOOL

Lectures on Advanced
Numerical Analysis

A series of lectures on advanced numerical
analysis presented at the University of Maryland
at College Park and recorded after the fact.

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