

Chapman & Hall/ CRC Mathematical Biology and Medicine Series

---

# DIFFERENTIAL EQUATIONS AND MATHEMATICAL BIOLOGY

D. S. JONES  
B. D. SLEEMAN



CHAPMAN & HALL/CRC

---

A CRC Press Company  
Boca Raton London New York Washington, D.C.

---

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Population growth	1
1.2	Administration of drugs	4
1.3	Cell division	6
1.4	Differential equations with separable variables	8
1.5	General properties	10
1.6	Equations of homogeneous type	12
1.7	Linear differential equations of the first order	14
1.8	Notes	17
	Exercises	17
<b>2</b>	<b>Linear Ordinary Differential Equations with Constant Coefficients</b>	<b>23</b>
2.1	Introduction	23
2.2	First-order linear differential equations	25
2.3	Linear equations of the second order	26
2.4	Finding the complementary function	27
2.5	Determining a particular integral	30
2.6	Forced oscillations	39
2.7	Differential equation of order $n$	41
2.8	Uniqueness	43
	Exercises	45
<b>3</b>	<b>Simultaneous Equations with Constant Coefficients</b>	<b>49</b>
3.1	Simultaneous equations of the first order	49
3.2	Replacement of one differential equation by a system	52
3.3	The general system	54
3.4	The fundamental system	56
3.5	Matrix notation	60
3.6	Initial and boundary value problems	66
3.7	Solving the inhomogeneous differential equation	70
3.8	Appendix: symbolic computation	71
	Exercises	79
<b>4</b>	<b>Modelling Biological Phenomena</b>	<b>83</b>
4.1	Introduction	83
4.2	Heart beat	83

4.3	Blood flow	86
4.4	Nerve impulse transmission	92
4.5	Chemical reactions	97
4.6	Predator-prey models	101
4.7	Notes	104
	Exercises	106
<b>5</b>	<b>First-order Systems of Ordinary Differential Equations</b>	<b>109</b>
5.1	Existence and uniqueness	109
5.2	Epidemics	112
5.3	The phase plane	113
5.4	Local stability	115
5.5	Stability	122
5.6	Limit cycles	127
5.7	Forced oscillations	131
5.8	Appendix: existence theory	134
5.9	Appendix: computing trajectories	140
	Exercises	141
<b>6</b>	<b>Mathematics of Heart Physiology</b>	<b>145</b>
6.1	The local model	145
6.2	The threshold effect	148
6.3	The phase plane analysis and the heart beat model	150
6.4	Physiological considerations of the heart beat cycle	153
6.5	A model of the cardiac pacemaker	155
6.6	Notes	157
	Exercises	157
<b>7</b>	<b>Mathematics of Nerve Impulse Transmission</b>	<b>159</b>
7.1	Excitability and repetitive firing	159
7.2	Travelling waves	167
7.3	Qualitative behaviour of travelling waves	169
7.4	Notes	172
	Exercises	173
<b>8</b>	<b>Chemical Reactions</b>	<b>175</b>
8.1	Wavefronts for the Belousov-Zhabotinskii reaction	175
8.2	Phase plane analysis of Fisher's equation	176
8.3	Qualitative behaviour in the general case	177
8.4	Notes	182
	Exercises	182
<b>9</b>	<b>Predator and Prey</b>	<b>183</b>
9.1	Catching fish	183
9.2	The effect of fishing	185

9.3	The Volterra-Lotka model . . . . .	187
	Exercises . . . . .	191
<b>10</b>	<b>Partial Differential Equations</b>	<b>195</b>
10.1	Characteristics for equations of the first order . . . . .	195
10.2	Another view of characteristics . . . . .	202
10.3	Linear partial differential equations of the second order . . . . .	204
10.4	Elliptic partial differential equations . . . . .	207
10.5	Parabolic partial differential equations . . . . .	211
10.6	Hyperbolic partial differential equations . . . . .	211
10.7	The wave equation . . . . .	212
10.8	Typical problems for the hyperbolic equation . . . . .	217
10.9	The Euler-Darboux equation . . . . .	222
	Exercises . . . . .	223
<b>11</b>	<b>Evolutionary Equations</b>	<b>227</b>
11.1	The heat equation . . . . .	227
11.2	Separation of variables . . . . .	231
11.3	Simple evolutionary equations . . . . .	238
11.4	Comparison theorems . . . . .	246
11.5	Notes . . . . .	258
	Exercises . . . . .	258
<b>12</b>	<b>Problems of Diffusion</b>	<b>263</b>
12.1	Diffusion through membranes . . . . .	263
12.2	Energy and energy estimates . . . . .	269
12.3	Global behaviour of nerve impulse transmissions . . . . .	274
12.4	Global behaviour in chemical reactions . . . . .	278
12.5	Turing diffusion driven instability and pattern formation . . . . .	282
12.6	Finite pattern forming domains . . . . .	292
12.7	Notes . . . . .	296
	Exercises . . . . .	297
<b>13</b>	<b>Bifurcation and Chaos</b>	<b>301</b>
13.1	Bifurcation . . . . .	301
13.2	Bifurcation of a limit cycle . . . . .	305
13.3	Discrete bifurcation . . . . .	307
13.4	Chaos . . . . .	314
13.5	Stability . . . . .	318
13.6	The Poincaré plane . . . . .	321
13.7	Averaging . . . . .	328
13.8	Appendix: programs . . . . .	333
	Exercises . . . . .	335

<b>14 Growth of Tumours</b>	<b>339</b>
14.1 Introduction	339
14.2 A mathematical model of tumour growth	342
14.3 A spherical tumour	345
14.4 Stability	349
14.5 Notes	351
Exercises	352
<b>15 Epidemics</b>	<b>355</b>
15.1 The Kermack-McKendrick model	355
15.2 Vaccination	357
15.3 An incubation model	358
15.4 Spreading in space	362
Exercises	368
<b>Answers to Exercises</b>	<b>371</b>