

Ian Anderson

A First Course in Discrete Mathematics

With 63 Figures



Springer

Ian Anderson, MA, PhD
Department of Mathematics, University of Glasgow, University Gardens,
Glasgow G12 8QW, UK

Cover illustration elements reproduced by kind permission of:

Aptech Systems, Inc., Publishers of the GAUSS Mathematical and Statistical System, 23804 S.E. Kent-Kangley Road, Maple Valley, WA 98038, USA. Tel: (206) 432 - 7855 Fax (206) 432 - 7832 email: info@aptech.com URL: www.aptech.com
American Statistical Association: Chance Vol 8 No 1, 1995 article by KS and KW Heiner 'Tree Rings of the Northern Shawangunks' page 32 fig. 2.
Springer-Verlag: Mathematica in Education and Research Vol 4 Issue 3 1995 article by Roman E Maeder, Beatrice Amrhein and Oliver Gloor 'Illustrated Mathematics: Visualization of Mathematical Objects' page 9 fig 11, originally published as a CD ROM 'Illustrated Mathematics' by TELOS: ISBN 978-1-85233-236-5, German edition by Birkhauser; ISBN 978-1-85233-236-5
Mathematica in Education and Research Vol 4 Issue 3 1995 article by Richard J Gaylord and Kazume Nishidate 'Traffic Engineering with Cellular Automata' page 35 fig 2. Mathematica in Education and Research Vol 5 Issue 2 1996 article by Michael Trott 'The Implicitization of a Trefoil Knot' page 14.
Mathematica in Education and Research Vol 5 Issue 2 1996 article by Lee de Cola 'Coins, Trees, Bars and Bells: Simulation of the Binomial Process' page 19 fig 3. Mathematica in Education and Research Vol 5 Issue 2 1996 article by Richard Gaylord and Kazume Nishidate 'Contagious Spreading' page 33 fig 1. Mathematica in Education and Research Vol 5 Issue 2 1996 article by Joe Buhler and Stan Wagon 'Secrets of the Madelung Constant' page 50 fig 1.

Springer Undergraduate Mathematics Series ISSN 1615-2085

British Library Cataloguing in Publication Data

Anderson, Ian, 1942-

A first course in discrete mathematics. - (Springer undergraduate mathematics series)

1. Computer science - Mathematics 2. Combinatorial analysis

I. Title

510

Library of Congress Cataloging-in-Publication Data

Anderson, Ian, 1942-

A first course in discrete mathematics / Ian Anderson.

p. cm. -- (Springer undergraduate mathematics series)

Includes bibliographical references and index.

ISBN 978-1-85233-236-5 ISBN 978-0-85729-315-2 (eBook)

DOI 10.1007/978-0-85729-315-2

1. Mathematics. 2. Computer Science - Mathematics. I. Title. II. Series.

QA39.2.A533 2000

510--dc21

00-063762

Apart from any fair dealing for the purposes of research or private study, or criticism or review, as permitted under the Copyright, Designs and Patents Act 1988, this publication may only be reproduced, stored or transmitted, in any form or by any means, with the prior permission in writing of the publishers, or in the case of reprographic reproduction in accordance with the terms of licences issued by the Copyright Licensing Agency. Enquiries concerning reproduction outside those terms should be sent to the publishers.

© Springer-Verlag London 2002

Originally published by Springer-Verlag London Limited in 2002

MyCopy version of the original edition 2002

The use of registered names, trademarks etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant laws and regulations and therefore free for general use.

The publisher makes no representation, express or implied, with regard to the accuracy of the information contained in this book and cannot accept any legal responsibility or liability for any errors or omissions that may be made.

Typesetting: Camera ready by the author and Michael Mackey

12/3830-54321 Printed on acid-free paper SPIN 10887527

www.springer.com/mycopy

Contents

1. Counting and Binomial Coefficients	1
1.1 Basic Principles	1
1.2 Factorials	2
1.3 Selections	3
1.4 Binomial Coefficients and Pascal's Triangle	6
1.5 Selections with Repetitions	10
1.6 A Useful Matrix Inversion	13
2. Recurrence	19
2.1 Some Examples	19
2.2 The Auxiliary Equation Method	23
2.3 Generating Functions	26
2.4 Derangements	28
2.5 Sorting Algorithms	32
2.6 Catalan Numbers	34
3. Introduction to Graphs	43
3.1 The Concept of a Graph	43
3.2 Paths in Graphs	46
3.3 Trees	47
3.4 Spanning Trees	50
3.5 Bipartite Graphs	52
3.6 Planarity	54
3.7 Polyhedra	60
4. Travelling Round a Graph	69
4.1 Hamiltonian Graphs	69
4.2 Planarity and Hamiltonian Graphs	71
4.3 The Travelling Salesman Problem	74

4.4	Gray Codes	76
4.5	Eulerian Graphs	78
4.6	Eulerian Digraphs	81
5.	Partitions and Colourings	89
5.1	Partitions of a Set	89
5.2	Stirling Numbers	91
5.3	Counting Functions	94
5.4	Vertex Colourings of Graphs	96
5.5	Edge Colourings of Graphs	99
6.	The Inclusion–Exclusion Principle	107
6.1	The Principle	107
6.2	Counting Surjections	112
6.3	Counting Labelled Trees	113
6.4	Scrabble	114
6.5	The Ménage Problem	115
7.	Latin Squares and Hall’s Theorem	121
7.1	Latin Squares and Orthogonality	121
7.2	Magic Squares	125
7.3	Systems of Distinct Representatives	127
7.4	From Latin Squares to Affine Planes	131
8.	Schedules and 1-Factorisations	137
8.1	The Circle Method	137
8.2	Bipartite Tournaments and 1-Factorisations of $K_{n,n}$	142
8.3	Tournaments from Orthogonal Latin Squares	145
9.	Introduction to Designs	149
9.1	Balanced Incomplete Block Designs	149
9.2	Resolvable Designs	156
9.3	Finite Projective Planes	159
9.4	Hadamard Matrices and Designs	161
9.5	Difference Methods	165
9.6	Hadamard Matrices and Codes	167
	Appendix	179
	Solutions	183
	Further Reading	195
	Bibliography	197
	Index	199