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Nonlinear Functional Analysis and its Applications

I: Fixed-Point Theorems

Translated by Peter R. Wadsack

With 158 Illustrations



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