

AN INTRODUCTION TO NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

J. DAVID LOGAN

*Department of Mathematics and Statistics
University of Nebraska-Lincoln
Lincoln, Nebraska*



A Wiley-Interscience Publication

JOHN WILEY & SONS, INC.

New York • Chichester • Brisbane • Toronto • Singapore

CONTENTS

Preface	ix
1. Partial Differential Equations	1
1.1. Partial Differential Equations, 2	
1.2. Conservation Laws, 13	
1.3. Constitutive Relations and Examples, 19	
1.4. Initial and Boundary Value Problems, 26	
1.5. Waves, 36	
References, 48	
2. First Order Equations and Characteristics	51
2.1. Linear First Order Equations, 51	
2.2. Nonlinear Equations, 56	
2.3. Quasilinear Equations, 60	
2.4. Propagation of Singularities, 65	
2.5. General First Order Equation, 71	
2.6. Uniqueness Result, 77	
References, 79	
3. Weak Solutions to Hyperbolic Equations	80
3.1. Discontinuous Solutions, 81	
3.2. Shock Formation, 92	
3.3. Traffic Flow; Plug Flow Reactors, 99	
3.4. Weak Solutions: A Formal Approach, 108	
3.5. Asymptotic Behavior of Shocks, 116	
References, 125	
4. Diffusion Processes	126
4.1. Review of Basic Results, 127	
4.2. Similarity Methods, 134	
4.3. Nonlinear Diffusion Models, 141	

4.4.	Reaction-Diffusion; Fisher's Equation, 151	
4.5.	Convection-Diffusion; Burgers' Equation, 161	
4.6.	Asymptotic Solutions to Burgers' Equation, 166	
	Appendix: Phase Plane, 173	
	References, 182	
5.	Hyperbolic Systems	184
5.1.	Waves in Shallow Water; Gas Dynamics, 184	
5.2.	Hyperbolic Systems and Characteristics, 194	
5.3.	Riemann's Method, 205	
5.4.	Hodographs; Wavefronts, 217	
5.5.	Weakly Nonlinear Approximations, 227	
	References, 233	
6.	Reaction-Diffusion Equations	234
6.1.	Reaction-Diffusion Models, 235	
6.2.	Traveling Wave Solutions, 244	
6.3.	Existence of Solutions, 257	
6.4.	Maximum Principles and Comparison Theorems, 274	
6.5.	Energy Estimates and Asymptotic Behavior, 282	
	References, 299	
7.	Models for Chemically Reacting Fluids	301
7.1.	Equations of Reactive Flow, 302	
7.2.	Reactive Flow Models, 315	
7.3.	Detonation Waves, 326	
7.4.	Combustion, 344	
	References, 354	
8.	Elliptic Equations	356
8.1.	Elliptic Equations and Models, 357	
8.2.	Basic Results for Elliptic Operators, 366	
8.3.	Eigenvalue Problems, 372	
8.4.	Stability and Bifurcation, 378	
	References, 394	
Index		397