THEORETICAL AERODYNAMICS

BY

L. M. MILNE-THOMSON, C.B.E.

EMERITUS PROFESSOR OF APPLIED MATHEMATICS, UNIVERSITY OF ARIZONA
EMERITUS PROFESSOR OF MATHEMATICS IN THE ROYAL NAVAL COLLEGE
PROFESSOR OF APPLIED MATHEMATICS IN BROWN UNIVERSITY
PROFESSOR IN THE MATHEMATICS RESEARCH CENTER AT THE UNIVERSITY OF WISCONSIN
VISITING PROFESSOR AT THE UNIVERSITIES OF ROME, QUEENSLAND, CALGARY, OTAGO

FOURTH EDITION
Revised and enlarged

DOVER PUBLICATIONS, INC.
NEW YORK

CONTENTS

PAGE

SYMBOLS xvii

REFERENCES TO LITERATURE xix

GREEK ALPHABET xx

CHAPTER I

PRELIMINARY NOTIONS

1·01. Aerodynamic force 1

1·02. Lift and drag 2

1·1. Monoplane aircraft 3

1·11. Chord of a profile 4

1·12. Chord of an aerofoil 5

1·13. Aspect ratio 5

1·14. Camber 6

1·15. Incidence 6

1·2. Fluids 7

1·21. Velocity 8

1·22. Streamlines and paths of particles 9

1·23. Stream tubes and filaments 10

1·3. Density 11

1·4. Pressure 11

1·41. Thrust due to pressure 12

1·5. The speed of sound 13

1·6. Maxwell's definition of viscosity 15

1·7. Physical dimensions 16

1·71. Aerodynamic force; dimensional theory 17

1·72. Similar systems; scale effect 19

1·73. Coefficients 19

1·8. The boundary layer 20

1·9. Approximations 23

EXAMPLES I 26

CHAPTER II

BERNOULLI'S THEOREM

2·1. Bernoulli's theorem 29

2·11. Incompressible fluid in the gravitational field 30

2·12. The constant in Bernoulli's theorem 31

2·13. Aerodynamic pressure 31

2·2. The Pitot tube 32

2·3. The work done by air in expanding 33

PAGE

2·31. Bernoulli's theorem for compressible flow - - - - - - - - 34
2·32. Application of Bernoulli's theorem to adiabatic expansion - - - - - 34
2·4. The Venturi tube - - - - - - - - - - - - - 36
2·41. Flow of air measured by the Venturi tube - - - - - - - - 37
2·5. Standard atmosphere - - - - - - - - - - - - 37
EXAMPLES II - - - - - - - - - - - - - 39

CHAPTER III

TWO-DIMENSIONAL MOTION

3·0. Motion in two dimensions - - - - - - - - - - - - 42
3·1. Stream function - - - - - - - - - - - - - 42
3·11. Velocity derived from the stream function - - - - - - - - - 44
3·12. Rankine's theorem - - - - - - - - - - - - 44
3·13. The stream function of a uniform wind - - - - - - - - - 45
3·14. Circular cylinder - - - - - - - - - - - - 46
3·15. The dividing streamline - - - - - - - - - - - - 47
3·2. Circulation - - - - - - - - - - - - - - 47
3·21. Vorticity - - - - - - - - - - - - - - 49
3·22. Motion of a fluid element - - - - - - - - - - - - 50
3·3. Irrotational motion - - - - - - - - - - - - - 51
3·31. Velocity potential - - - - - - - - - - - - - 52
3·311. Laplace's equation - - - - - - - - - - - - - 53
3·32. Cyclic motion - - - - - - - - - - - - - - 54
3·4. Complex numbers - - - - - - - - - - - - - 54
3·41. The argument - - - - - - - - - - - - - - 56
3·42. Differentiation - - - - - - - - - - - - - - 57
3·43. Holomorphic functions - - - - - - - - - - - - - 57
3·44. Conjugate functions - - - - - - - - - - - - - 57
3·45. The function $\overline{f}(z)$ - - - - - - - - - - - - - 58
3·47. The coordinates z and $\bar{z}$ - - - - - - - - - - - - 59
3·5. Cauchy's integral theorem - - - - - - - - - - - - 60
3·51. Singularities - - - - - - - - - - - - - - 60
3·52. Residues - - - - - - - - - - - - - - - 61
3·53. Cauchy's residue theorem - - - - - - - - - - - - - 61
3·6. Conformal mapping - - - - - - - - - - - - - 62
3·7. Complex potential - - - - - - - - - - - - - 63
3·71. The complex velocity - - - - - - - - - - - - - 65
3·8. Application of conformal mapping - - - - - - - - - - - 66
EXAMPLES III - - - - - - - - - - - - - - 67

CHAPTER IV

RECTILINEAR VORTICES

4·0. Two-dimensional vortices - - - - - - - - - - - - 70
4·1. Circular vortex - - - - - - - - - - - - - - 70

PAGE

4·11. Velocity distribution 72
4·12. Size of a circular vortex 73
4·2. Point rectilinear vortex 74
4·21. Vortex pair 74
4·22. Image of a vortex in a plane 75
4·3. Vortex between parallel planes 76
4·4. Force on a vortex 77
4·5. Mutual action of two vortices 77
4·6. Energy due to a pair of vortices 78
4·7. Continuous line of vortices 79
4·71. Evaluation of the definite integral 80
EXAMPLES IV 81

CHAPTER V

THE CIRCULAR CYLINDER AS AN AEROFOIL

5·0. Preliminary 83
5·1. The points z and a^2/z 83
5·2. The circle theorem 84
5·3. Circulation about a circular cylinder 85
5·31. Circular cylinder in a wind with circulation 86
5·311. Given stagnation point 87
5·32. The pressure on the cylinder 88
5·33. Force on the cylinder 89
5·4. The theorem of Blasius 89
5·41. Theorem of Blasius in terms of the stream function 91
5·5. The theorem of Kutta and Joukowski 91
5·7. The second circle theorem 93
5·72. Uniform shear flow 93
5·74. Circular cylinder in uniform shear flow 94
EXAMPLES V 95

CHAPTER VI

JOUKOWSKI'S TRANSFORMATION

6·1. Joukowski's transformation 97
6·11. Circles with centre at the origin 98
6·2. Joukowski fins, rudders, and struts 99
6·3. Circular arc profiles 99
6·4. The general Joukowski profile 100
6·5. Geometrical construction 101
6·51. Mapping a circle 102
6·6. Reversal 105
6·7. Construction of tangents 106
6·8. The airflow 106
EXAMPLES VI 108

CHAPTER VII

THEORY OF TWO-DIMENSIONAL AEROFOILS

		PAGE
7·0.	Types of profile	111
7·01.	Conditions to be satisfied	111
7·02.	Origin at the centre of the circle	113
7·03.	Some properties of profiles obtained by transforming a circle	114
7·1.	Aerodynamic force	114
7·11.	Joukowski's hypothesis	115
7·12.	The lift	116
7·13.	Lift coefficient	116
7·14.	Pitching moment coefficient	118
7·2.	Focus of a profile	120
7·21.	Metacentric parabola	121
7·3.	Centre of pressure	122
7·31.	Centre of pressure of a Joukowski rudder	122
7·4.	Centroid of the circulation	123
7·41.	The third axis of a profile	124
7·5.	Force at a sharp point of a profile	125
7·51.	The flat aerofoil	127
7·6.	Kármán-Trefftz profiles	128
7·7.	Von Mises profiles	131
7·8.	Carafoli profiles	134
	EXAMPLES VII	134

CHAPTER VIII

THIN AEROFOILS

8·0.	Geometry of profiles	136
8·01.	Thin aerofoils of small camber	138
8·1.	The flat aerofoil	138
8·2.	The general problem	140
8·3.	Glauert's method of solution	141
8·31.	Calculation of the induced velocity	141
8·32.	Determination of the coefficients A_n	141
8·33.	The lift	142
8·34.	The pitching moment	144
8·35.	Travel of the centre of pressure	145
8·36.	Unsymmetrical aerofoil with a fixed centre of pressure	145
8·37.	Effect of operating a flap	146
8·4.	The pressure method	147
8·41.	The boundary problem	148
8·42.	The inverse problem	151
8·43.	Determination of a camber line	151
8·44.	Determination of a thickness function	154
8·5.	Substitution vortex	156

PAGE

8·6. The two-dimensional biplane 158
8·7. Approximate theory of the biplane 159
8·8. Equal biplane 160
EXAMPLES VIII 162

CHAPTER IX

INDUCED VELOCITY

9·1. Vector notation 165
9·2. The equation of motion 166
9·21. Kelvin's circulation theorem 166
9·3. Vorticity 167
9·31. Vortex lines, tubes, and filaments 168
9·4. The law of induced velocity 169
9·5. Velocity induced by a vortex filament 171
9·51. Straight filament 171
9·52. The horseshoe vortex 172
9·6. Vortex sheet 173
9·61. Velocity of a point of a vortex sheet 175
9·63. Application to aerofoils 176
EXAMPLES IX 177

CHAPTER X

AEROFOIL OF FINITE ASPECT RATIO

10·1. Steady motion 180
10·2. The generation of the vortex system 182
10·21. The impulse of a vortex ring 184
10·3. The lift 185
10·4. Instability of the vortex sheet 185
10·41. Tip vortices 186
10·5. The velocity of the air 186
10·51. The velocity at a point of the aerofoil 187
10·6. Aerodynamic force 188
10·61. The force **F** 189
10·7. Moments 190

CHAPTER XI

THE LIFTING LINE THEORY

11·0. Preliminary 191
11·01. Geometrical hypotheses 191
11·1. Axes of reference 191
11·2. Expression for the lift 192
11·201. Distribution of load 194
11·21. Expression for the induced drag 194

PAGE

11·22. The downwash velocity at the trailing edge - - - - - - - - 196
11·23. The loading law - - - - - - - - - - - - - 197
11·24. Effect of downwash on incidence - - - - - - - - 197
11·3. The integral equation for the circulation - - - - - - - 200
11·4. Problem I; elliptic loading - - - - - - - - 200
11·41. Elliptic loading; lift and induced drag - - - - - - - 202
11·42. Slope of the lift graph for elliptic loading - - - - - - - 203
11·43. Change of aspect ratio in elliptic loading - - - - - - - 204
11·5. Problem II - - - - - - - - - - - - 204
11·51. The lift - - - - - - - - - - - - 207
11·52. The downwash velocity - - - - - - - - - - 207
11·53. The induced drag - - - - - - - - - - - 208
11·6. Minimum induced drag - - - - - - - - - - 209
11·7. The wake far downwind - - - - - - - - - - 209
11·8. Lift and drag deduced from the impulse - - - - - - - 210
11·9. The rectangular aerofoil - - - - - - - - - - 211
11·91. Cylindrical rectangular aerofoil - - - - - - - - - 211
11·92. Method of Betz - - - - - - - - - - - 211
11·93. Method of Fuchs - - - - - - - - - - - 213
EXAMPLES XI - - - - - - - - - - - - 213

CHAPTER XII

LIFTING SURFACE THEORY

12·1. Velocity induced by a lifting line element - - - - - - - - 216
12·11. Munk's theorem of stagger - - - - - - - - - - 218
12·12. The induced lift - - - - - - - - - - - - 219
12·2. Blenk's method - - - - - - - - - - - - 220
12·21. Rectangular aerofoil - - - - - - - - - - - 221
12·22. Calculation of the downwash velocity - - - - - - - - 222
12·23. Side-slipping rectangular aerofoil - - - - - - - - - 224
12·3. Aerofoils of small aspect ratio - - - - - - - - - 225
12·31. The integral equation - - - - - - - - - - - 226
12·32. Zero aspect ratio - - - - - - - - - - - 227
12·4. The acceleration potential - - - - - - - - - - 228
12·41. Lifting surface - - - - - - - - - - - - 229
12·5. Aerofoil of circular plan - - - - - - - - - - 230
12·6. Aerofoil of elliptic plan - - - - - - - - - - - 231
EXAMPLES XII - - - - - - - - - - - - 232

CHAPTER XIII

PROPELLERS

13·1. Propellers - - - - - - - - - - - - - 233
13·2. How thrust is developed - - - - - - - - - - - 234

PAGE
13·3. The slipstream - - - - - - - - - - - - - 235
13·31. Velocity and pressure in the slipstream - - - - - - - - 237
13·4. Interference velocity - - - - - - - - - - - - 238
13·41. The force on a blade element - - - - - - - - - - 239
13·42. Characteristic coefficients - - - - - - - - - - - 240
13·5. Infinitely many blades - - - - - - - - - - - 242
13·51. The enclosed propeller - - - - - - - - - - - 243
13·6. Froude's law - - - - - - - - - - - - - 244
13·7. Interference factors - - - - - - - - - - - - 244
EXAMPLES XIII - - - - - - - - - - - - - 245

CHAPTER XIV

WIND TUNNEL CORRECTIONS

14·0. Preliminary - - - - - - - - - - - - - 247
14·1. Nature of the corrections - - - - - - - - - - - 248
14·2. Boundary conditions - - - - - - - - - - - - 248
14·3. Reduction to two dimensions - - - - - - - - - - 249
14·31. Axes of reference - - - - - - - - - - - - 250
14·4. Circular closed section - - - - - - - - - - - 250
14·41. Elliptic loading - - - - - - - - - - - - 252
14·42. Open jet circular section - - - - - - - - - - - 253
14·43. Glauert's theorem - - - - - - - - - - - - 253
14·5. Parallel plane boundaries - - - - - - - - - - - 255
14·6. The general problem - - - - - - - - - - - - 256
14·7. Jacobian elliptic functions - - - - - - - - - - - 258
14·8. Rectangular section - - - - - - - - - - - - 262
14·9. Mapping an ellipse on a circle - - - - - - - - - - 264
14·91. Elliptic section - - - - - - - - - - - - - 266
EXAMPLES XIV - - - - - - - - - - - - - 267

CHAPTER XV

SUBSONIC FLOW

15·0. Preliminary - - - - - - - - - - - - - 270
15·01. Thermodynamical considerations - - - - - - - - - - 270
15·1. Steady homentropic flow - - - - - - - - - - - 272
15·2. Irrotational motion - - - - - - - - - - - - 274
15·3. Linear perturbations - - - - - - - - - - - - 275
15·4. Linearised subsonic flow - - - - - - - - - - - 276
15·41. Distortion of the streamlines - - - - - - - - - - 277
15·42. Two-dimensional flow - - - - - - - - - - - 279
15·43. Lifting line - - - - - - - - - - - - - 281
15·44. The hodograph method - - - - - - - - - - - 281

PAGE
15·45. The hodograph equations for homentropic flow 283
15·46. Velocity correction factor 284
Examples XV 286

CHAPTER XVI

SUPERSONIC FLOW

16·1. Moving disturbance 289
16·15. Thrust due to a supersonic jet 290
16·2. Linearised supersonic flow 291
16·3. Flow round a corner 293
16·32. Characteristics 295
16·35. Prandtl-Meyer expansion 296
16·36. Complete Prandtl-Meyer expansion 298
16·37. Inversion of the deflection equation 301
16·4. Shock waves 302
16·45. The shock polar 304
16·46. Critical angle 307
16·6. The flat aerofoil 308
16·61. Flat aerofoil with flap 310
16·7. Flow past a polygonal profile 311
16·71. Flow past a general profile 311
Examples XVI 314

CHAPTER XVII

SUPERSONIC SWEPTBACK AND DELTA WINGS

17·01. Conical flow 317
17·02. Rectangular aerofoil 317
17·03. Two-dimensional sweep-back 319
17·1. Linearisation for the flat aerofoil 320
17·12. Solution of the linearised equation for conical flow 322
17·13. The complex variable ϵ 324
17·14. Complex velocity components 326
17·15. Compatibility equations 326
17·16. Polygonal aerofoils 328
17·17. Boundary conditions 331
17·18. Circular boundary value problems 333
17·19. Non-lifting aerofoils 336
17·2. Force on a delta wing 338
17·3. Planform I_0I_0 340
17·31. Planform I_nI_n 342
17·4. Delta wing of constant lift distribution. 342

Cambridge University Press www.cambridge.org

PAGE
17·41. Planform L_yT_x - - - - - - - - - - - - - - 344
17·5. Planform L_yA - - - - - - - - - - - - - - 346
17·51. Planform L_yA - - - - - - - - - - - - - - 349
17·52. Planform L_yL_i - - - - - - - - - - - - - - 350
17·53. Planform L_yT_i - - - - - - - - - - - - - - 354
17·6. Planform L_iT_i - - - - - - - - - - - - - - 356
17·61. Determination of the constant B - - - - - - - - - - 357
17·62. Planform L_iL_i - - - - - - - - - - - - - - 359
17·63. Determination of the constant C - - - - - - - - - - 360
17·64. A limit case of planform L_iL_i - - - - - - - - - - 361
17·65. Planform L_iA - - - - - - - - - - - - - - 362
17·7. Interference of conical flows - - - - - - - - - - 363
EXAMPLES XVII - - - - - - - - - - - - - - 364

CHAPTER XVIII

SIMPLE FLIGHT PROBLEMS

18·0. Preliminary - - - - - - - - - - - - - - 368
18·1. Linear flight - - - - - - - - - - - - - - 369
18·2. Stalling - - - - - - - - - - - - - - - 369
18·3. Gliding - - - - - - - - - - - - - - - 372
18·31. Straight horizontal flight - - - - - - - - - - - 373
18·32. Sudden increase of incidence - - - - - - - - - - 374
18·33. Straight side-slip - - - - - - - - - - - - - 374
18·4. Banked turn - - - - - - - - - - - - - - 375
18·5. Lanchester's phugoids - - - - - - - - - - - - 376
18·51. The phugoid oscillation - - - - - - - - - - - - 378
EXAMPLES XVIII - - - - - - - - - - - - - - 378

CHAPTER XIX

MOMENTS

19·0. Pitching moment - - - - - - - - - - - - - - 379
19·1. Static stability - - - - - - - - - - - - - - 379
19·11. Metacentric ratio - - - - - - - - - - - - - 380
19·12. Neutral centre of gravity positions - - - - - - - - - 381
19·2. Asymmetric moments - - - - - - - - - - - - - 383
19·3. The strip hypothesis - - - - - - - - - - - - - 383
19·4. Moments due to rolling - - - - - - - - - - - - 384
19·5. Moments due to yawing - - - - - - - - - - - - 385
19·6. Rolling moment due to side-slip - - - - - - - - - - 386
19·7. Change of axes - - - - - - - - - - - - - - 387
EXAMPLES XIX - - - - - - - - - - - - - - 388

CHAPTER XX

STABILITY

PAGE
20·0. Preliminary 389
20·01. Equations of motion 389
20·1. Straight flight 390
20·11. Simplifying assumption 392
20·12. The equations of disturbed horizontal flight 393
20·2. The parameter μ 394
20·21. Units 394
20·3. Expression in non-dimensional form 395
20·4. Dynamical stability 397
20·5. Longitudinal stability 397
20·51. Routh's discriminant 398
20·52. Method of Gates 399
20·6. Laplace transform 400
20·61. Method of solution 401
20·62. Sudden application of elevators 402
20·63. Approximate solution of the quartic 403
EXAMPLES XX 404

CHAPTER XXI

VECTORS

21·1. Scalars and vectors 406
21·11. Resolution of vectors 407
21·12. The scalar product of two vectors 408
21·13. The vector product of two vectors 409
21·14. Equations of motion of an aircraft 411
21·2. Triple products 412
21·3. Vector differentiation ; the operator nabla 413
21·31. The acceleration of an air particle 415
21·4. Gauss's theorem 415
21·5. The equation of continuity 417
21·6. The equation of motion 418
21·7. Stokes's theorem 418
EXAMPLES XXI 420
INDEX 423