

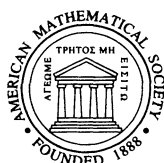
Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 125

**Theory of
Commutative Fields**

Masayoshi Nagata

Translated by
Masayoshi Nagata



American Mathematical Society
Providence, Rhode Island

Contents

Preface to the English Edition	xi
Preface to the New Japanese Edition	xiii
Preface to the Original Japanese Edition	xv
Chapter 0. Prerequisites from Set Theory	1
§0. Basic symbols	1
§1. Mappings	1
§2. Ordered sets	3
§3. Partitions and equivalence relations	4
Chapter I. Groups, Rings, and Fields	7
§1. Groups	7
§2. Normal subgroups and homomorphisms	10
§3. Rings and fields	16
§4. Integral domains and prime ideals	20
§5. Polynomial rings	23
§6. Unique factorization	25
§7. Modules	29
§8. Symmetric forms and alternating forms	34
Exercises	37
Chapter II. Algebraic Extensions of Finite Degrees	43
§1. Basic notions	43
§2. Splitting fields	47
§3. Separability and inseparability	48
§4. Multiplicative groups of finite fields	51
§5. Simple extensions	51
§6. Normal extensions	54
§7. Invariants of a finite group	55
§8. The fundamental theorem of Galois	58
§9. Roots of unity and cyclic extensions	59
§10. Solvability of algebraic equations	63
§11. Construction problems	67

§12. Algebraically closed fields	71
Appendix 1	73
Appendix 2	74
Exercises	75
Chapter III. Transcendental Extensions	81
§1. Transcendence bases	81
§2. Tensor products over a field	83
§3. Derivations	87
§4. Separable extensions	91
§5. Regular extensions	94
§6. Noetherian rings	96
§7. Integral extensions and prime ideals	102
§8. The normalization theorem for polynomial rings	107
§9. Integral closures	111
§10. Algebraic varieties	113
§11. The C_i -conditions	118
§12. The theorem of Lüroth	122
Appendix. A theorem on valuation rings and its applications	124
Exercises	126
Chapter IV. Valuations	135
§1. Multiplicative valuations	135
§2. Valuations of the rational number field	138
§3. Topology	140
§4. Topological groups and topological fields	145
§5. Completions	149
§6. Archimedean valuations and absolute values	152
§7. Additive valuations and valuation rings	154
§8. Approximation theorems	159
§9. Prolongations of a valuation	162
§10. The product formula	168
§11. Hensel's lemma	170
Exercises	182
Chapter V. Formally Real Fields	191
§1. Ordered fields, formally real fields, and real closed fields	191
§2. Real closures	196
§3. Hilbert's 17th Problem	202
§4. A valuation corresponding to an order	206
Exercises	211

Chapter VI. Galois Theory of Algebraic Extensions of Infinite Degree	215
§1. Topology on a Galois group	215
§2. The fundamental theorem of Galois	217
§3. Splitting fields, inertia fields, and ramification fields	219
§4. Algebraic equations of high degrees	221
Exercises	224
Answers and Hints	227
Index of Symbols	243
Subject Index	245