

Volume 1 / Fundamental Algorithms

**THE ART OF
COMPUTER PROGRAMMING
SECOND EDITION**

Reading, Massachusetts • Menlo Park, California
Don Mills, Ontario • Wokingham, England • Amsterdam • Bonn • Sydney
Singapore • Tokyo • Madrid • Bogota • Santiago • San Juan

CONTENTS

Chapter 1—Basic Concepts	1
1.1. Algorithms	1
1.2. Mathematical Preliminaries	10
1.2.1. Mathematical Induction	11
1.2.2. Numbers, Powers, and Logarithms	21
1.2.3. Sums and Products	26
1.2.4. Integer Functions and Elementary Number Theory	37
1.2.5. Permutations and Factorials	44
1.2.6. Binomial Coefficients	51
1.2.7. Harmonic Numbers	73
1.2.8. Fibonacci Numbers	78
1.2.9. Generating Functions	86
1.2.10. Analysis of an Algorithm	94
*1.2.11. Asymptotic Representations	104
*1.2.11.1. The O -notation	104
*1.2.11.2. Euler's summation formula	108
*1.2.11.3. Some asymptotic calculations	112
1.3. MIX	120
1.3.1. Description of MIX	120
1.3.2. The MIX Assembly Language	141
1.3.3. Applications to Permutations	160
1.4. Some Fundamental Programming Techniques	182
1.4.1. Subroutines	182
1.4.2. Coroutines	190
1.4.3. Interpretive Routines	197
1.4.3.1. A MIX simulator	198
*1.4.3.2. Trace routines	208
1.4.4. Input and Output	211
1.4.5. History and Bibliography	225
Chapter 2—Information Structures	228
2.1. Introduction	228
2.2. Linear Lists	234
2.2.1. Stacks, Queues, and Deques	234
2.2.2. Sequential Allocation	240
2.2.3. Linked Allocation	251
2.2.4. Circular Lists	270

2.2.5.	Doubly Linked Lists	278
2.2.6.	Arrays and Orthogonal Lists	295
2.3.	Trees	305
2.3.1.	Traversing Binary Trees	315
2.3.2.	Binary Tree Representation of Trees	332
2.3.3.	Other Representations of Trees	347
2.3.4.	Basic Mathematical Properties of Trees	362
2.3.4.1.	Free trees	362
*2.3.4.2.	Oriented trees	371
*2.3.4.3.	The "infinity lemma"	381
*2.3.4.4.	Enumeration of trees	385
2.3.4.5.	Path length	399
*2.3.4.6.	History and bibliography	405
2.3.5.	Lists and Garbage Collection	406
2.4.	Multilinked Structures	423
2.5.	Dynamic Storage Allocation	435
2.6.	History and Bibliography	456
	Answers to Exercises	465
	Appendix A—Index to Notations	607
	Appendix B—Tables of Numerical Quantities	
1.	Fundamental Constants (decimal)	613
2.	Fundamental Constants (octal)	614
3.	Harmonic Numbers, Bernoulli Numbers, Fibonacci Numbers	615
	Index and Glossary	617