

IRVING KAPLANSKY

University of Chicago

AN INTRODUCTION TO
DIFFERENTIAL
ALGEBRA

Second edition, revised, corrected and enlarged



HERMANN

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