

From Kant to Hilbert:
A Source Book in the
Foundations of
Mathematics

Volume I

WILLIAM EWALD
The Law School
University of Pennsylvania

CLARENDON PRESS · OXFORD

1996

CONTENTS

Volume I

Copyright Permissions	xvii
Introduction	1
1. GEORGE BERKELEY (1685–1753)	11
A. <i>From the Philosophical commentaries</i> <i>(Berkeley 1707–8)</i>	13
B. Of infinites <i>(Berkeley 1901 [1707])</i>	16
C. Letter to Samuel Molyneux <i>(Berkeley 1709)</i>	19
D. <i>From A treatise concerning the principles of</i> <i>human knowledge, Part One</i> <i>(Berkeley 1710)</i>	21
E. De Motu <i>(Berkeley 1721)</i>	37
F. <i>From Alciphron</i> <i>(Berkeley 1732)</i>	54
G. <i>From Newton's Principia mathematica</i> <i>(Newton 1726)</i>	58
H. The analyst <i>(Berkeley 1734)</i>	60
2. COLIN MACLAURIN (1698–1746)	93
A. <i>From A treatise of fluxions</i> <i>(MacLaurin 1742)</i>	95
3. JEAN LE ROND D'ALEMBERT (1717–1783)	123
A. Differential <i>(D'Alembert 1754)</i>	123
B. Infinite <i>(D'Alembert 1765a)</i>	128
C. Limit <i>(D'Alembert 1765b)</i>	130

4.	IMMANUEL KANT (1724–1804)	132
A.	<i>From Thoughts on the true estimation of active forces (Kant 1747)</i>	133
B.	<i>From the Transcendental aesthetic (Kant 1787)</i>	135
C.	<i>From the Discipline of pure reason (Kant 1781)</i>	136
D.	<i>Frege on Kant (Frege 1884)</i>	148
5.	JOHANN HEINRICH LAMBERT (1728–1777)	152
A.	<i>From the Theory of parallel lines (Lambert 1786)</i>	158
6.	BERNARD BOLZANO (1781–1848)	168
A.	<i>Preface to Considerations on some objects of elementary geometry (Bolzano 1804)</i>	172
B.	<i>Contributions to a better-grounded presentation of mathematics (Bolzano 1810)</i>	174
C.	<i>Purely analytic proof of the theorem that between any two values which give results of opposite sign there lies at least one real root of the equation (Bolzano 1817a)</i>	225
D.	<i>From Paradoxes of the infinite (Bolzano 1851)</i>	249
7.	CARL FRIEDRICH GAUSS (1777–1855)	293
A.	<i>On the metaphysics of mathematics (Gauss 1929)</i>	293
B.	<i>Gauss on non-Euclidean geometry</i>	296
C.	<i>Notice on the theory of biquadratic residues (Gauss 1831)</i>	306
8.	DUNCAN GREGORY (1813–1844)	314
A.	<i>On the real nature of symbolical algebra (Gregory 1840)</i>	323

9.	AUGUSTUS DE MORGAN (1806–1871)	331
	A. On the foundation of algebra (<i>De Morgan 1842a</i>)	336
	B. Trigonometry and double algebra (<i>De Morgan 1849b</i>)	349
10.	WILLIAM ROWAN HAMILTON (1805–1865)	362
	A. <i>From the Theory of conjugate functions, or algebraic couples; with a preliminary and elementary essay on algebra as the science of pure time</i> (<i>Hamilton 1837</i>)	369
	B. <i>Preface to the Lectures on quaternions</i> (<i>Hamilton 1853</i>)	375
	C. <i>From the Correspondence of Hamilton with De Morgan</i>	425
11.	GEORGE BOOLE (1815–1864)	442
	A. The mathematical analysis of logic, being an essay towards a calculus of deductive reasoning (<i>Boole 1847</i>)	451
12.	JAMES JOSEPH SYLVESTER (1814–1897)	510
	A. Presidential address to Section 'A' of the British Association (<i>Sylvester 1869</i>)	511
13.	WILLIAM KINGDON CLIFFORD (1845–1879)	523
	A. On the space theory of matter (<i>Clifford 1876</i>)	523
	B. On the aims and instruments of scientific thought (<i>Clifford 1872</i>)	524
14.	ARTHUR CAYLEY (1821–1895)	542
	A. Presidential address to the British Association, September 1883 (<i>Cayley 1883</i>)	542

15.	CHARLES SANDERS PEIRCE (1839–1914)	574
A.	<i>From</i> Linear associative algebra (<i>Benjamin Peirce 1870</i>)	584
B.	Notes on Benjamin Peirce's linear associative algebra (<i>Peirce 1976</i>)	594
C.	On the logic of number (<i>Peirce 1881</i>)	596
D.	On the algebra of logic: a contribution to the philosophy of notation (<i>Peirce 1885</i>)	608
E.	The logic of mathematics in relation to education (<i>Peirce 1898</i>)	632
F.	<i>From</i> The simplest mathematics (<i>Peirce 1902</i>)	637
	References to Volume I	I
	Index to Volume I	XXIII

Volume II

16.	GEORG FRIEDRICH BERNHARD RIEMANN (1826–1866)	649
A.	On the hypotheses which lie at the foundation of geometry (<i>Riemann 1868</i>)	652
17.	HERMANN VON HELMHOLTZ (1821–1894)	662
A.	The origin and meaning of geometrical axioms (<i>Helmholtz 1876a</i>)	663
B.	The facts in perception (<i>Helmholtz 1878b</i>)	689
C.	Numbering and measuring from an epistemological viewpoint (<i>Helmholtz 1887</i>)	727

18.	JULIUS WILHELM RICHARD DEDEKIND (1831–1916)	753
A.	On the introduction of new functions in mathematics (<i>Dedekind 1854</i>)	754
B.	<i>From the Tenth Supplement to Dirichlet's Lectures on the theory of numbers</i> (<i>Dedekind 1871</i>)	762
C.	Continuity and irrational numbers (<i>Dedekind 1872</i>)	765
D.	<i>From On the theory of algebraic integers</i> (<i>Dedekind 1877</i>)	779
E.	Was sind und was sollen die Zahlen? (<i>Dedekind 1888</i>)	787
F.	<i>From the Eleventh Supplement to Dirichlet's Lectures on the theory of numbers</i> (<i>Dedekind 1894</i>)	833
G.	Letter to Heinrich Weber (24 January 1888)	834
H.	Felix Bernstein on Dedekind and Cantor	836
I.	<i>From the Nachlass</i>	836
19.	GEORG CANTOR (1845–1918)	838
A.	On a property of the set of real algebraic numbers (<i>Cantor 1874</i>)	839
B.	The early correspondence between Cantor and Dedekind	843
C.	Foundations of a general theory of manifolds: a mathematico-philosophical investigation into the theory of the infinite (<i>Cantor 1883d</i>)	878
D.	On an elementary question in the theory of manifolds (<i>Cantor 1891</i>)	920
E.	Cantor's late correspondence with Dedekind and Hilbert	923
20.	LEOPOLD KRONECKER (1823–1891)	941
A.	Hilbert and Kronecker (<i>From Weyl 1944b</i>)	942
B.	Extract from Hilbert's Göttingen lectures	943

C.	Two footnotes (From <i>Kronecker 1881</i> and <i>1886</i>)	946
D.	On the concept of number (<i>Kronecker 1887</i>)	947
21.	CHRISTIAN FELIX KLEIN (1849–1925)	956
A.	Klein on the schools of mathematics (From <i>Klein 1911</i>)	957
B.	On the mathematical character of space-intuition and the relation of pure mathematics to the applied sciences (From <i>Klein 1911</i>)	958
C.	The arithmetizing of mathematics (<i>Klein 1895</i>)	965
22.	JULES HENRI POINCARÉ (1854–1912)	972
A.	On the nature of mathematical reasoning (<i>Poincaré 1894</i>)	972
B.	On the foundations of geometry (<i>Poincaré 1898</i>)	982
C.	Intuition and logic in mathematics (<i>Poincaré 1900</i>)	1012
D.	Mathematics and logic: I (<i>Poincaré 1905b</i>)	1021
E.	Mathematics and logic: II (<i>Poincaré 1906a</i>)	1038
F.	Mathematics and logic: III (<i>Poincaré 1906b</i>)	1052
G.	On transfinite numbers (<i>Poincaré 1910</i>)	1071
23.	THE FRENCH ANALYSTS	1075
A.	Some remarks on the principles of the theory of sets (<i>Borel 1905</i>)	1076
B.	Five letters on set theory (<i>Baire et alii 1905</i>)	1077
24.	DAVID HILBERT (1862–1943)	1087
A.	On the concept of number (<i>Hilbert 1900a</i>)	1089
B.	From Mathematical problems (<i>Hilbert 1900b</i>)	1096

C.	Axiomatic thought (<i>Hilbert 1918</i>)	1105
D.	The new grounding of mathematics First report (<i>Hilbert 1922a</i>)	1115
E.	The logical foundations of mathematics (<i>Hilbert 1923a</i>)	1134
F.	The grounding of elementary number theory (<i>Hilbert 1931a</i>)	1148
G.	Logic and the knowledge of nature (<i>Hilbert 1930b</i>)	1157
25.	LUITZEN EGBERTUS JEAN BROUWER (1881–1966)	1166
A.	Mathematics, science, and language (<i>Brouwer 1928a</i>)	1170
B.	The structure of the continuum (<i>Brouwer 1928b</i>)	1186
C.	Historical background, principles, and methods of intuitionism (<i>Brouwer 1952</i>)	1197
26.	ERNST ZERMELO (1871–1953)	1208
A.	On boundary numbers and domains of sets: new investigations in the foundations of set theory (<i>Zermelo 1930</i>)	1219
27.	GODFREY HAROLD HARDY (1877–1947)	1234
A.	Sir George Stokes and the concept of uniform convergence (<i>Hardy 1918</i>)	1234
B.	Mathematical proof (<i>Hardy 1929a</i>)	1243
28.	NICOLAUS BOURBAKI	1264
A.	The architecture of mathematics (<i>Bourbaki 1948</i>)	1265
	Bibliography	1277
	Index	1331