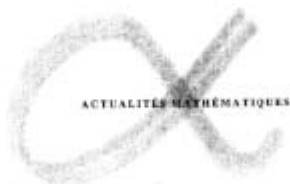


Alexandru Buium

Differential Algebra
and
Diophantine Geometry



HERMANN  ÉDITEURS DES SCIENCES ET DES ARTS

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