

FIRST COURSE
in
FUNCTIONAL ANALYSIS

CASPER GOFFMAN

*Professor of Mathematics
Purdue University*

GEORGE PEDRICK

*Professor of Mathematics
California State University, Hayward*

CHELSEA PUBLISHING COMPANY
NEW YORK, N.Y. 10016

CONTENTS

Chapter 1—Metric Space

1

1.1	Definitions and Examples	1
1.2	Inequalities of Hölder and Minkowski	2
1.3	Examples Continued; l_p Spaces	4
1.4	Examples Continued; Function Spaces	6
1.5	Convergence and Related Notions	8
1.6	Separable Space, Examples	9
1.7	Complete Space, Examples	11
1.8	Contractions, Applications to Differential and Integral Equations	16
1.9	Completion	20
1.10	Category, Nowhere Differentiable Continuous Functions	21
1.11	Compactness, Continuity	24
1.12	Equicontinuity, Application to Differential Equations	28
1.13	Stone-Weierstrass Theorems	32
1.14	Normal Families	35
1.15	Semi-continuity, Application to Arc Length	38
1.16	Space of Compact, Convex Sets	41
	Exercises	43

Chapter 2—Banach Spaces

50

2.1	Vector Space	50
2.2	Subspace	52
2.3	Quotient Space	54
2.4	Dimension, Hamel Basis	55
2.5	Algebraic Dual, Second Dual	57
2.6	Convex Sets	58
2.7	Ordered Groups	59
2.8	Hahn-Banach Theorem, Separation Form	60
2.9	Hahn-Banach Theorem, Extension Form	62
2.10	Applications, Banach Limits, Invariant Measure	64
2.11	Banach Space, Dual Space	71
2.12	Hahn-Banach Theorem in Normed Space	73
2.13	Uniform Boundedness Principle, Applications	76

2.14 Lemma of F. Riesz, Applications	81
2.15 Application to Compact Transformations	83
2.16 Applications, Weak Convergence, Summability Methods, Approximate Integration	86
2.17 Second Dual Space	91
2.18 Dual of l_p	93
2.19 Dual of $C[a, b]$, Riesz Representation Theorem	94
2.20 Open Mapping and Closed Graph Theorems	97
2.21 Application, Projections	100
2.22 Application, Schauder Expansion	101
2.23 A Theorem on Operators in $C[0, 1]$	102
Exercises	103

Chapter 3—Measure and Integration, L_p Spaces

109

3.1 Lebesgue Measure for Bounded Sets in E_n	109
3.2 Lebesgue Measure for Unbounded Sets	113
3.3 Totally σ Finite Measures	114
3.4 Measurable Functions, Egoroff Theorem	115
3.5 Convergence in Measure	118
3.6 Summable Functions	120
3.7 Fatou and Lebesgue Dominated Convergence Theorems	125
3.8 Integral as a Set Function	128
3.9 Signed Measure, Decomposition into Measures	129
3.10 Absolute Continuity and Singularity of Measures	132
3.11 The L_p Spaces, Completeness	135
3.12 Approximation and Smoothing Operations	138
3.13 The Dual of L_p , $p > 1$	142
3.14 The Dual of L_1	146
3.15 The Individual Ergodic Theorem	148
3.16 L_p Convergence of Fourier Series	150
3.17 Functions Whose Fourier Series Diverge Almost Everywhere	153
3.18 Continuous Functions Which Differ from All Those Having a Given Modulus	155
Exercises	159

Chapter 4—Hilbert Space

165

4.1 Inner Product, Hilbert Space	165
4.2 Basic Lemma, Projection Theorem, Dual	168
4.3 Application, Mean Ergodic Theorem	171
4.4 Orthonormal Sets, Fourier Expansion	173
4.5 Application, Isoperimetric Theorem	178
4.6 Müntz Theorem	179
4.7 Dimension, Riesz-Fischer Theorem	182
4.8 Reproducing Kernel	184
4.9 Application, Bergman Kernel	187

4.10 Examples of Complete Orthonormal Sets	191
4.11 Systems of Haar, Rademacher, Walsh; Applications	194
Exercises	201
Chapter 5—Topological Vector Spaces	206
5.1 Topology	206
5.2 Tychonoff Theorem, Application in Banach Space	207
5.3 Topological Vector Space	210
5.4 Normable Space	211
5.5 Space of Measurable Functions	212
5.6 Locally Convex Space	217
5.7 Metrizable Space, Space of Entire Functions	219
5.8 FK Spaces	224
5.9 Application to Summability Methods	228
5.10 Ordered Vector Spaces	232
5.11 Banach Lattice	235
5.12 Köthe Spaces	238
Exercises	242
Chapter 6—Banach Algebras	248
6.1 Definition and Examples	248
6.2 Adjunction of Identity	250
6.3 Haar Measure	250
6.4 Commutative Banach Algebras, Maximal Ideals	254
6.5 The Set $C(\mathcal{M})$	258
6.6 Gelfand Representation for Algebras with Identity	260
6.7 Analytic Functions	261
6.8 Isomorphism Theorem for Algebras with Identity	263
6.9 Algebras without Identity	266
6.10 Application to $L_1(G)$	269
Exercises	272
References	276
Index	279