

LINEAR SPACES AND DIFFERENTIATION THEORY

ALFRED FRÖLICHER

University of Geneva, Switzerland

ANDREAS KRIEGL

University of Vienna, Austria

A Wiley-Interscience Publication

JOHN WILEY & SONS

Chichester · New York · Brisbane · Toronto · Singapore

CONTENTS

<i>Preface</i>	ix
1 Foundational Material	1
1.1 Categories generated by a set of maps	2
1.2 Bornologies and ℓ^∞ -structures	6
1.3 Difference quotients	12
1.4 Lipschitz structures and smooth structures	22
2 Convenient Vector Spaces	27
2.1 Locally convex and convex bornological spaces	28
2.2 The Mackey convergence and the Mackey closure topology	35
2.3 ℓ^∞ -vector-spaces and $\mathcal{L}(\beta^k)$ -vector-spaces	41
2.4 Preconvenient vector spaces	47
2.5 Separation	52
2.6 Completion	55
3 Multilinear Maps and Categorical Properties	60
3.1 Initial and final structures, categorical completeness	61
3.2 Subspaces and quotients	63
3.3 Products	65
3.4 Coproducts or direct sums	68
3.5 Inductive limits	70
3.6 Function spaces of linear maps	71
3.7 Function spaces of multilinear maps	75
3.8 The tensor product	77
3.9 The duality functor	79
4 Calculus in Convenient Vector Spaces	81
4.1 Differentiable curves	83
4.2 Curve spaces	93

4.3	Differentiable maps	99	
4.4	Function spaces and exponential laws	114	
4.5	Partial differentiability	134	
4.6	Spaces of sections of vector bundles	138	
4.7	Certain function spaces are manifolds	145	
4.8	Theorems on inverse and implicit functions	151	
5	Differentiable Maps and Categorical Properties		154
5.1	Free convenient vector spaces	155	
5.2	Convenient co-algebras	165	
5.3	Cartesian closed categories of convenient vector spaces		173
5.4	Reflexivity	179	
6	The Mackey Closure Topology		188
6.1	Comparison with other topologies	189	
6.2	Continuity of the addition and regularity	192	
6.3	The Mackey closure of subsets	197	
6.4	Convex functions	199	
7	Permanence Properties and Counter-examples		204
7.1	Extension and lifting properties	205	
7.2	Preservation of categorical limits	209	
7.3	Preservation of initiality and finality	216	
7.4	Various counter-examples	221	
8	Some Categorical Notions and Notations		224
8.1	Categories	224	
8.2	Functors and natural transformations	225	
8.3	Limits and colimits	226	
8.4	Adjoint functors	228	
8.5	Adjoint functors and limits	229	
8.6	Cartesian closed categories	229	
8.7	Initial sources and final sinks	230	
8.8	Embeddings and quotient maps	232	
	<i>Bibliography</i>		234
	<i>List of Symbols</i>		236
	<i>List of Categories</i>		239
	<i>List of Functors</i>		240
	<i>Index</i>		242