ELEMENTS OF THE THEORY OF FUNCTIONS

BY KONRAD KNOPP

Professor of Mathematics at the University of Tübingen

TRANSLATED BY FREDERICK BAGEMIHL

DOVER PUBLICATIONS, INC.

CONTENTS

SECTION I. COMPLEX NUMBERS AND THEIR GEOMETRIC REPRESENTATION

Chapter I. Foundations	
1. Introduction	1
2. The system of real numbers	2
3. Points and vectors of the plane	7
Chapter II. The System of Complex Numbers and the Gaussian Plane of Numbers	
4. Historical remarks	13
5. Introduction of complex numbers. Notation	15
6. Equality and inequality	19
7. Addition and subtraction	20
8. Multiplication and division	22
9. Derived rules. Powers	24
10. The system of complex numbers as an extension of the system	
of real numbers	25
11. Trigonometric representation of complex numbers	27
 Geometric representation of multiplication and division 	30
13. Inequalities and absolute values. Examples	31
Chapter III. The Riemann Sphere of Numbers	
14. The stereographic projection	35
 The Riemann sphere of numbers. The point ∞. Examples 	39
SECTION II. LINEAR FUNCTIONS AND CIRCULAR TRANSPORMATIONS	
Chapter IV. Mapping by Means of Linear Functions	
16. Mapping by means of entire linear functions	48
17. Mapping by means of the function $w = 1/z$	45
18. Mapping by means of arbitrary linear functions	50
Chapter V. Normal Forms and Particular Linear Mappings	
19. The group-property of linear transformations	58
20. Fixed points and normal forms	54
21. Particular linear mappings. Cross ratios	59
22. Further examples	62

SECTION III. SETS AND SEQUENCES. POWER SERIES				
Chapter VI. Point Sets and Sets of Numbers				
23. Point sets				
24. Sets of real numbers	68			
25. The Bolzano-Weierstrass theorem	70			
Chapter VII. Sequences of Numbers. Infinite Series				
26. Sequences of complex numbers				
27. Sequences of real numbers				
28. Infinite series	76			
Chapter VIII. Power Series				
29. The circle of convergence	83			
30. Operations on power series	86			
SECTION IV. ANALYTIC FUNCTIONS AND CONFORMAL MAPPING				
Chapter IX. Functions of a Complex Variable				
31. The concept of a function of a complex variable	89			
32. Limits of functions	90			
83. Continuity	92			
34. Differentiability	93			
35. Properties of functions represented by power series	96			
Chapter X. Analytic Functions and Conformal Mapping				
36. Analytic functions	101			
37. Conformal mapping	102			
SECTION V. THE ELEMENTARY FUNCTIONS				
Chapter XI. Power and Root. The Rational Functions				
38. Power and root	106			
39. The entire rational functions	110			
40. The fractional rational functions	111			
Chapter XII. The Exponential, Trigonometric, and				
Hyperbolic Functions				
41. The exponential function	113 119			
42. The functions cos z and sin z				

123

125

45. The functions tan z and cot z

44. The hyperbolic functions

Chapter X	III. The	Logar	rithm,	the	Cyclometric	Func-
	tion	s, and	the E	linor	nial Series	

45. The logarithm	127
48. The cyclometric functions	130
47. The binomial series and the general power	133
Bibliography	136
Index	157