Ordinary Differential Equations

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PREFACE

The material in this book is not a conventional treatment of ordinary differential equations. It does not contain the collection of proofs commonly displayed as the foundations of the subject, nor does it contain the collection of recipes commonly aimed at the scientist or engineer. Rather, in a way which requires little or no previous acquaintance with the subject, it contains a sequence of heuristic arguments, illustrative examples and exercises, which serve to lead the reader towards the invention, generalization, and usage of the techniques by which solutions of differential equations can be constructed. Above all, we hope, the reader can gain a perspective concerning the extent to which methods which lead *in principle* to the solution of a given problem actually lead to a *useful* description of that solution.

Our purpose is to offer an alternative to the almost "rote" approach, in which the standard categories of differential equations, accompanied by routine problem sets, are systematically listed. We firmly believe that the present approach is one that should be encountered, at least once, by mathematicians, users of mathematics, and those who are merely curious about mathematics; we hope that members of all three sets will find the presentation stimulating.

We consider the exercises to be an essential part of the text. They extend, amplify, and provide perspective for the text material. (In rare cases, the statement of a problem is left deliberately incomplete, so as to give the reader some scope for ingenuity.) If this book is used to accompany a course of lectures, one technique would be to assign text reading and exercises beforehand, to be followed by classroom discussion and amplification. Alternatively, special problem sessions could be included.

In reprinting this book, we have taken the opportunity to correct an (embarrassingly large!) number of misprints, and to clarify certain aspects of the presentation. As in the original printing, the attitudes and approaches in this book remain solely the responsibility of the authors; however, we gratefully acknowledge the efforts of the many colleagues who have struggled to reform our viewpoints. In this connection, we are particularly appreciative of the suggestions provided by Bob Margulies and Frank Fendell, each of whom was kind enough to read portions of the present manuscript.

G.F.C.
C.E.P.

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