

Martin Braun

Differential Equations and Their Applications

An Introduction to
Applied Mathematics

Fourth Edition

With 68 Illustrations

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