

An Introduction to
**Quantum
Field
Theory**

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THE ADVANCED BOOK PROGRAM



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an informa business

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*These sections may be omitted in a one-year course emphasizing the less formal aspects of elementary particle physics.