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The Stability of Dynamical Systems

Appendix A Limiting Equations and Stability of Nonautonomous Ordinary Differential Equations

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PHILADELPHIA, PENNSYLVANIA 1976

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