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Ideals, Varieties, and Algorithms

An Introduction to Computational Algebraic
Geometry and Commutative Algebra

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