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Dale Husemoller

Fibre Bundles

Third Edition

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To the memory of my mother and my father

Preface to the Third Edition

In this edition, we have added two new chapters, Chapter 7 on the gauge group of a principal bundle and Chapter 19 on the definition of Chern classes by differential forms. These subjects have taken on special importance when we consider new applications of the fibre bundle theory especially to mathematical physics. For these two chapters, the author profited from discussions with Professor M. S. Narasimhan.

The idea of using the term bundle for what is just a map, but is eventually a fibre bundle projection, is due to Grothendieck.

The bibliography has been enlarged and updated. For example, in the Seifert reference [1932] we find one of the first explicit references to the concept of fibrings.

The first edition of the *Fibre Bundles* was translated into Russian under the title "Расслоенные Пространства" in 1970 by B. A. Йсковских with general editor M. M. Постникова. The remarks and additions of the editor have been very useful in this edition of the book. The author is very grateful to A. Voronov, who helped with translations of the additions from the Russian text.

Part of this revision was made while the author was a guest of the Max Planck Institut from 1988 to 89, the ETH during the summers of 1990 and 1991, the University of Heidelberg during the summer of 1992, and the Tata Institute for Fundamental Research during January 1990, 1991, and 1992. It is a pleasure to acknowledge all these institutions as well as the Haverford College Faculty Research Fund.

1993 Dale Husemoller

Preface to the Second Edition

In this edition we have added a section to Chapter 15 on the Adams conjecture and a second appendix on the suspension theorems. For the second appendix the author profitted from discussion with Professors Moore, Stasheff, and Toda.

I wish to express my gratitude to the following people who supplied me with lists of corrections to the first edition: P. T. Chusch, Rudolf Fritsch, David C. Johnson, George Lusztig, Claude Schocket, and Robert Sturg.

Part of the revision was made while the author was a guest of the I.H.E.S in January, May, and June 1974.

1974 Dale Husemoller

Preface to the First Edition

The notion of a fibre bundle first arose out of questions posed in the 1930s on the topology and geometry of manifolds. By the year 1950, the definition of fibre bundle had been clearly formulated, the homotopy classification of fibre bundles achieved, and the theory of characteristic classes of fibre bundles developed by several mathematicians: Chern, Pontrjagin, Stiefel, and Whitney. Steenrod's book, which appeared in 1950, gave a coherent treatment of the subject up to that time.

About 1955, Milnor gave a construction of a universal fibre bundle for any topological group. This construction is also included in Part I along with an elementary proof that the bundle is universal.

During the five years from 1950 to 1955, Hirzebruch clarified the notion of characteristic class and used it to prove a general Riemann-Roch theorem for algebraic varieties. This was published in his Ergebnisse Monograph. A systematic development of characteristic classes and their applications to manifolds is given in Part III and is based on the approach of Hirzebruch as modified by Grothendieck.

In the early 1960s, following lines of thought in the work of A. Grothendieck, Atiyah and Hirzebruch developed K-theory, which is a generalized cohomology theory defined by using stability classes of vector bundles. The Bott periodicity theorem was interpreted as a theorem in K-theory, and J. F. Adams was able to solve the vector field problem for spheres, using K-theory. In Part II, an introduction to K-theory is presented, the nonexistence of elements of Hopf invariant 1 proved (after a proof of Atiyah), and the proof of the vector field problem sketched.

I wish to express gratitude to S. Eilenberg, who gave me so much encouragement during recent years, and to J. C. Moore, who read parts of the

manuscript and made many useful comments. Conversations with J. F. Adams, R. Bott, A. Dold, and F. Hirzebruch helped to sharpen many parts of the manuscript. During the writing of this book, I was particularly influenced by the Princeton notes of J. Milnor and the lectures of F. Hirzebruch at the 1963 Summer Institute of the American Mathematical Society.

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