

Grundlehren der mathematischen Wissenschaften 243

A Series of Comprehensive Studies in Mathematics

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Finite Groups III



Springer-Verlag
Berlin Heidelberg New York 1982

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Library of Congress Cataloging in Publication Data. Huppert, Bertram, 1927–.
Finite groups III. (Grundlehren der mathematischen Wissenschaften; 243).
Bibliography: p. Includes index. I. Finite groups. I. Blackburn, N. (Norman).
II. Title. III. Series. QA 171.B 578. 512'.22. 81-2288.
ISBN-13: 978-3-642-67999-5 e-ISBN-13: 978-3-642-67997-1
DOI: 10.1007/978-3-642-67997-1

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© Springer-Verlag Berlin Heidelberg 1982
Softcover reprint of the hardcover 1st edition 1982

Typesetting: Asco Trade Typesetting Limited, Chai Wan, Hong Kong.

2141/3140-543210

Preface



From "Die Meistersinger von Nürnberg", Richard Wagner

This final volume is concerned with some of the developments of the subject in the 1960's. In attempting to determine the simple groups, the first step was to settle the conjecture of Burnside that groups of odd order are soluble. The proof that this conjecture was correct is much too long and complicated for presentation in this text, but a number of ideas in the early stages of it led to a local theory of finite groups, some aspects of which are discussed in Chapter X. Much of this discussion is a continuation of the theory of the transfer (see Chapter IV), but we also introduce the generalized Fitting subgroup, which played a basic role in characterization theorems, that is, in descriptions of specific groups in terms of group-theoretical properties alone. One of the earliest and most important such characterizations was given for Zassenhaus groups; this is presented in Chapter XI. Characterizations in terms of the centralizer of an involution are of particular importance in view of the theorem of Brauer and Fowler. In Chapter XII, one such theorem is given, in which the Mathieu group \mathfrak{M}_{11} and $PSL(3, 3)$ are characterized. This last chapter is mainly concerned with some aspects of multiply transitive permutation groups loosely connected with the Mathieu groups or with sharp n -fold transitivity, and several results from Chapter XI are used in it. The two last chapters are, however, independent of Chapter X.

Again we wish to acknowledge our indebtedness to the many colleagues who have assisted us with this work. In addition to those named in the preface to Volume II, thanks are due to George Glauberman, who read an earlier version of Chapter X. The contributions of all have done a great deal to improve this volume, and it is with the greatest pleasure that we express our gratitude to them.

January, 1982

Bertram Huppert, Mainz
Norman Blackburn, Manchester

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