Graduate Texts in Mathematics **9**

Springer
New York
Berlin
Heidelberg
Hong Kong
London
Milan
Paris
Tokyo

Graduate Texts in Mathematics

1 TAKEUTI/ZARING. Introduction to Axiomatic Set Theory. 2nd ed.
2 OXTOBY. Measure and Category. 2nd ed.
3 SCHAEFER. Topological Vector Spaces. 2nd ed.
4 HILTON/STAMMBACH. A Course in Homological Algebra. 2nd ed.
5 MAC LANE. Categories for the Working Mathematician. 2nd ed.
6 HUGHES/PIPER. Projective Planes.
7 SERRE. A Course in Arithmetic.
8 TAKEUTI/ZARING. Axiomatic Set Theory.
9 HUMPHREYS. Introduction to Lie Algebras and Representation Theory.
10 COHEN. A Course in Simple Homotopy Theory.
11 CONWAY. Functions of One Complex Variable I. 2nd ed.
12 BEALS. Advanced Mathematical Analysis.
13 ANDERSON/FULLER. Rings and Categories of Modules. 2nd ed.
14 GOLUBITSKY/GUILLEMIN. Stable Mappings and Their Singularities.
15 BERBERIAN. Lectures in Functional Analysis and Operator Theory.
16 WINTER. The Structure of Fields.
17 ROSENBLATT. Random Processes. 2nd ed.
18 HALMOS. Measure Theory.
19 HALMOS. A Hilbert Space Problem Book. 2nd ed.
20 HUSEMOLLER. Fibre Bundles. 3rd ed.
21 HUMPHREYS. Linear Algebraic Groups.
22 BARNES/MACK. An Algebraic Introduction to Mathematical Logic.
23 GREUB. Linear Algebra. 4th ed.
24 HOLMES. Geometric Functional Analysis and Its Applications.
25 HEWITT/STROMBERG. Real and Abstract Analysis.
26 MANES. Algebraic Theories.
27 KELLEY. General Topology.
28 ZARISKI/SAMUEL. Commutative Algebra. Vol.I.
29 ZARISKI/SAMUEL. Commutative Algebra. Vol.II.
30 JACOBSON. Lectures in Abstract Algebra I. Basic Concepts.
31 JACOBSON. Lectures in Abstract Algebra II. Linear Algebra.
32 JACOBSON. Lectures in Abstract Algebra III. Theory of Fields and Galois Theory.
33 HIRSCH. Differential Topology.
34 SPITZER. Principles of Random Walk. 2nd ed.
35 ALEXANDER/WERMER. Several Complex Variables and Banach Algebras. 3rd ed.
36 KELLEY/NAMIOKA et al. Linear Topological Spaces.
37 MONK. Mathematical Logic.
38 GRAUERT/FRITZSCHE. Several Complex Variables.
39 ARVESON. An Invitation to C^*-Algebras.
40 KEMENY/SNELL/KNAPP. Denumerable Markov Chains. 2nd ed.
41 APOSTOL. Modular Functions and Dirichlet Series in Number Theory. 2nd ed.
42 SERRE. Linear Representations of Finite Groups.
43 GILLMAN/JERISON. Rings of Continuous Functions.
44 KENDIG. Elementary Algebraic Geometry.
45 LOÈVE. Probability Theory I. 4th ed.
46 LOÈVE. Probability Theory II. 4th ed.
47 MOISE. Geometric Topology in Dimensions 2 and 3.
48 SACHS/WU. General Relativity for Mathematicians.
49 GRUENBERG/WEIR. Linear Geometry. 2nd ed.
50 EDWARDS. Fermat's Last Theorem.
51 KLINGENBERG. A Course in Differential Geometry.
52 HARTSHORNE. Algebraic Geometry.
53 MANIN. A Course in Mathematical Logic.
54 GRAVER/WATKINS. Combinatorics with Emphasis on the Theory of Graphs.
55 BROWN/PEARCY. Introduction to Operator Theory I: Elements of Functional Analysis.
56 MASSEY. Algebraic Topology: An Introduction.
57 CROWELL/FOX. Introduction to Knot Theory.
58 KOBLITZ. p-adic Numbers, p-adic Analysis, and Zeta-Functions. 2nd ed.
59 LANG. Cyclotomic Fields.
60 ARNOLD. Mathematical Methods in Classical Mechanics. 2nd ed.
61 WHITEHEAD. Elements of Homotopy Theory.
62 KARGAPOLOV/MERLZJAKOV. Fundamentals of the Theory of Groups.
63 BOLLOBAS. Graph Theory.
64 EDWARDS. Fourier Series. Vol. I. 2nd ed.
65 WELLS. Differential Analysis on Complex Manifolds. 2nd ed.

(continued after index)

James E. Humphreys

Introduction to Lie Algebras and Representation Theory

Springer

J. E. Humphreys
Professor of Mathematics
University of Massachusetts
Amherst, MA 01003
USA

Mathematics Subject Classification (2000): 17BXX, 20G05, 22E60

ISBN-13: 978-0-387-90052-0 e-ISBN-13: 978-1-4612-6398-2
DOI: 10.1007/978-1-4612-6398-2

Printed on acid-free paper.

Library of Congress Catalog Card Number 72-85951

19 18 17 16 15 14 13 12 11 10 SPIN 11011378

Springer-Verlag is a part of *Springer Science+Business Media*

springeronline.com

To the memory of my nephews

Willard Charles Humphreys III

and

Thomas Edward Humphreys

Preface

This book is designed to introduce the reader to the theory of semisimple Lie algebras over an algebraically closed field of characteristic 0, with emphasis on representations. A good knowledge of linear algebra (including eigenvalues, bilinear forms, euclidean spaces, and tensor products of vector spaces) is presupposed, as well as some acquaintance with the methods of abstract algebra. The first four chapters might well be read by a bright undergraduate; however, the remaining three chapters are admittedly a little more demanding.

Besides being useful in many parts of mathematics and physics, the theory of semisimple Lie algebras is inherently attractive, combining as it does a certain amount of depth and a satisfying degree of completeness in its basic results. Since Jacobson's book appeared a decade ago, improvements have been made even in the classical parts of the theory. I have tried to incorporate some of them here and to provide easier access to the subject for non-specialists. For the specialist, the following features should be noted:

(1) The Jordan-Chevalley decomposition of linear transformations is emphasized, with "toral" subalgebras replacing the more traditional Cartan subalgebras in the semisimple case.

(2) The conjugacy theorem for Cartan subalgebras is proved (following D. J. Winter and G. D. Mostow) by elementary Lie algebra methods, avoiding the use of algebraic geometry.

(3) The isomorphism theorem is proved first in an elementary way (Theorem 14.2), but later obtained again as a corollary of Serre's Theorem (18.3), which gives a presentation by generators and relations.

(4) From the outset, the simple algebras of types A, B, C, D are emphasized in the text and exercises.

(5) Root systems are treated axiomatically (Chapter III), along with some of the theory of weights.

(6) A conceptual approach to Weyl's character formula, based on Harish-Chandra's theory of "characters" and independent of Freudenthal's multiplicity formula (22.3), is presented in §23 and §24. This is inspired by D.-N. Verma's thesis, and recent work of I. N. Bernstein, I. M. Gel'fand, S. I. Gel'fand.

(7) The basic constructions in the theory of Chevalley groups are given in Chapter VII, following lecture notes of R. Steinberg.

I have had to omit many standard topics (most of which I feel are better suited to a second course), e.g., cohomology, theorems of Levi and Mal'cev, theorems of Ado and Iwasawa, classification over non-algebraically closed fields, Lie algebras in prime characteristic. I hope the reader will be stimulated to pursue these topics in the books and articles listed under References, especially Jacobson [1], Bourbaki [1], [2], Winter [1], Seligman [1].

A few words about mechanics: Terminology is mostly traditional, and notation has been kept to a minimum, to facilitate skipping back and forth in the text. After Chapters I–III, the remaining chapters can be read in almost any order if the reader is willing to follow up a few references (except that VII depends on §20 and §21, while VI depends on §17). A reference to Theorem 14.2 indicates the (unique) theorem in subsection 14.2 (of §14). Notes following some sections indicate nonstandard sources or further reading, but I have not tried to give a history of each theorem (for historical remarks, cf. Bourbaki [2] and Freudenthal-deVries [1]). The reference list consists largely of items mentioned explicitly; for more extensive bibliographies, consult Jacobson [1], Seligman [1]. Some 240 exercises, of all shades of difficulty, have been included; a few of the easier ones are needed in the text.

This text grew out of lectures which I gave at the N.S.F. Advanced Science Seminar on Algebraic Groups at Bowdoin College in 1968; my intention then was to enlarge on J.-P. Serre's excellent but incomplete lecture notes [2]. My other literary debts (to the books and lecture notes of N. Bourbaki, N. Jacobson, R. Steinberg, D. J. Winter, and others) will be obvious. Less obvious is my personal debt to my teachers, George Seligman and Nathan Jacobson, who first aroused my interest in Lie algebras. I am grateful to David J. Winter for giving me pre-publication access to his book, to Robert L. Wilson for making many helpful criticisms of an earlier version of the manuscript, to Connie Engle for her help in preparing the final manuscript, and to Michael J. DeRise for moral support. Financial assistance from the Courant Institute of Mathematical Sciences and the National Science Foundation is also gratefully acknowledged.

New York, April 4, 1972 J. E. Humphreys

Notation and Conventions

Z, $\mathbf{Z}^+$, **Q**, **R**, **C** denote (respectively) the integers, nonnegative integers, rationals, reals, and complex numbers

$\amalg$ denotes direct sum of vector spaces

$A \rtimes B$ denotes the semidirect product of groups A and B, with B normal

Card	= cardinality	Ker	= kernel
char	= characteristic	Im	= image
det	= determinant	Tr	= trace
dim	= dimension		

Table of Contents

Introduction to Lie Algebras and Representation Theory