Handbook of Multivalued Analysis

Mathematics and Its Applications

Volume 500

Handbook of Multivalued Analysis

Volume II: Applications

by

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SPRINGER-SCIENCE+BUSINESS MEDIA, B.V.

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-1-4613-7111-3 ISBN 978-1-4615-4665-8 (eBook)
DOI 10.1007/978-1-4615-4665-8

Printed on acid-free paper

Dedication

To the memory of my father Shiye Hu

To the memory of my father Socrates Papageorgiou

Contents

Preface

In volume I we developed the tools of "Multivalued Analysis." In this volume we examine the applications. After all, the initial impetus for the development of the theory of set-valued functions came from its applications in areas such as control theory and mathematical economics. In fact, the needs of control theory, in particular the study of systems with a priori feedback, led to the systematic investigation of differential equations with a multivalued vector field (differential inclusions). For this reason, we start this volume with three chapters devoted to set-valued differential equations. However, in contrast to the existing books on the subject (i.e. J.-P. Aubin - A. Cellina: "Differential Inclusions," Springer-Verlag, 1983, and Deimling: "Multivalued Differential Equations," W. De Gruyter, 1992), here we focus on "Evolution Inclusions," which are evolution equations with multivalued terms. Evolution equations were raised to prominence with the development of the linear semigroup theory by Hille and Yosida initially, with subsequent important contributions by Kato, Phillips and Lions. This theory allowed a successful unified treatment of some apparently different classes of nonstationary linear partial differential equations and linear functional equations. The needs of dealing with applied problems and the natural tendency to extend the linear theory to the nonlinear case led to the development of the nonlinear semigroup theory, which became a very effective tool in the analysis of broad classes of nonlinear evolution equations. This nonlinear breakthrough was achieved with the advent of the theory of operators of monotone type.

In the first two chapters, we deal with two large classes of evolution inclusions. The first class involves operators of monotone type which are coercive and defined in the context of an evolution triple. The second class involves subdifferential operators defined on Hilbert spaces and is suitable for the analysis of systems with unilateral constraints. The third chapter is a pot-pouri of topics from the theory of differential and evolution inclusions. Among other things, we discuss second order nonlinear multivalued boundary value problems and problems with discontinuities, which have been the focus of intense research activity in the last decade. After the chapters on the multivalued differential equations, in chapter 4 we turn to optimal control problems for infinite dimensional systems. Since most lumped parameters systems (finite dimensional systems) are approximations of distributed parameter systems, the study of infinite dimensional systems, such as control systems governed by partial differential equations or functional equations, are both of intrinsic interest and important in a great variety of applications.

We treat systems driven by elliptic, parabolic and evolution equations, and we go beyond optimal control theory by examining the controllability properties of nonlinear systems using the tools of operator theory. Optimal control theory grew out of the theory of calculus of variations. Calculus of variations is a subject as old as calculus itself. Together with optimal control theory and optimization theory, calculus of variations spans a large area of mathematical analysis, known as "theory of extremal problems." Recently there has been a renewal of interest in the classical problems, in particular in vectorial problems. Chapter 5 surveys some of these developments and underscores the significant role that multivalued analysis has in the investigation of variational problems. In chapter 6 we pass to a related class of dynamic optimization problems, which represent models of economic dynamics and the intertemporal distribution of resources. We consider both deterministic and stochastic models, in discrete and continuous time. Primarily, we are interested in two types of results: existence theorems for "optimal" programs and theorems characterizing such programs via a system of prices. It is important to stress that the models considered here are extreme idealizations. In this way we are able to build a sufficiently complete mathematical theory. Nevertheless, the main ideas and methods pervade the broad spectrum of concrete economic problems. The built-in "uncertainty" in the evolution of such systems leads naturally to set-valued dynamics and so, the theory of multivalued analysis plays a prominent role. At the end of the chapter (section 6.7) we discuss static models and in particular we investigate some central issues of modern equilibrium theory.

Chapter 7 stands at the interface of probability theory, optimal control theory, and mathematical economics. Probability theory began in the seventeenth century (at approximately the same time with calculus). However, only in the middle of this century, did mathematicians develop the tools to deal with the problems of maximizing the chances of winning against an intelligent opponent. These tools stand in the core of modern stochastic control and stochastic games. Chapter 7 surveys some aspects of these theories. Finally, in chapter 8 we have gathered various applications from optimization theory and mathematical economics, in which multivalued analysis and its byproducts are indispensable tools.

Needless to say, the list of applications is not exhaustive. Moreover, the same is true of our treatment of each subject. There are certainly several omissions which the experienced reader will spot easily. Nevertheless, we hope that with this volume have given a pretty good idea of the many applications that the tools and methods of multivalued analysis have found in the last four decades. With the information this volume provides, the reader will be able to pursue in greater detail and depth the study of the particular applications and models that may interest him. Closing this project, we can safely claim that the message is loud and clear: "Multivalued Analysis" is a very prolific branch of modern mathematical analysis, with fascinating theory and numerous important applications in physical, economic, and engineering problems.

It is a great pleasure to acknowledge a debt of personal gratitude to those who helped us with this difficult project. The first author wishes to express his deep gratitude to his high-school mathematical teacher Mr. Chaoqian Zong who brought him to the kingdom of mathematics; to Professors Yubo Chen and Wang Zhuang who helped him build a sound background in analysis; to Professor Klaus Deimling who introduced to him the fascinating world of the multivalued; and to Professor V. Lakshmikantham who helped him in diverse fields ranging from mathematics to philosophy. He further thanks Professors Lynn Erbe and Mitsuharu Otani for inviting him to their institutions to present talks purporting to this project.

The second author wishes to thank F. DeBlasi who was a constant source of inspiration and encouragement. He considers himself very fortunate to have a friend of the intellectual and moral caliber of F. DeBlasi. He is also indebted to his very good friends and collaborators from Perugia (Italy): A. Fiacca, F. Papalini, T. Cardinali; and from Krakow (Poland): Z. Denkowski, S. Migorski, L. Gasinski. The frequent visits in those two beautiful cities gave him the opportunity to present and develop parts of the material of this volume. Moreover, he would like to record his dept to his close friends and colleagues in Greece: E. Avgerinos, Ch. Koukouvinos, M. Mitrouli, N. Halidias, N. Kourogenis, and N. Yannakakis, for their continuous support. Finally, many thanks are also due to N.U. Ahmed, S. Aizicovici, R. Aliprantis, A. Kartsatos, U. Ledzewicz, and G. Pianigiani.

June 1999

Springfield, MO — Shouchuan Hu

Athens, — Nikolas S. Papageorgiou