Jens Høyrup

Lengths, Widths, Surfaces

A Portrait of Old Babylonian Algebra and Its Kin

With 89 Illustrations

Springer

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