Steven Homer Alan L. Selman

COMPUTABILITY AND COMPLEXITY THEORY

With 17 Illustrations

Springer

Steven Homer
Department of Computer Science
Boston University
111 Cummington Street
Boston, MA 02215, USA
homer@cs.bu.edu

Alan L. Selman
Department of Computer Science and Engineering
226 Bell Hall
University at Buffalo
Buffalo, NY 14260-2000, USA
selman@cse.buffalo.edu

Series Editors

David Gries
Department of Computer Science
415 Boyd Studies Research Center
The University of Georgia
Athens, GA 30605, USA

Fred B. Schneider
Department of Computer Science
Upson Hall
Cornell University
Ithaca, NY 14853-7501, USA

Library of Congress Cataloging-in-Publication Data
Homer, S. (Steven)
Computability and complexity theory / Steven Homer, Alan L. Selman.
p. cm. — (Texts in computer science)
Includes bibliographical references and index.
ISBN 978-1-4419-2865-8 ISBN 978-1-4757-3544-4 (eBook)
DOI 10.1007/978-1-4757-3544-4
1. Computer science. 2. Computable functions. 3. Computational complexity.
I. Selman, Alan L. II. Title. III. Series.
QA76 .H6236 2001
004—dc21 00-053829

Printed on acid-free paper.

© 2001 Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. in 2001
Softcover reprint of the hardcover 1st edition 2001

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by MaryAnn Brickner; manufacturing supervised by Joe Quatela.
Typeset pages prepared using the authors' LaTeX 2_ε files by Integre Technical Publishing Company, Inc.

9 8 7 6 5 4 3 2 1 SPIN 10769347

Contents

Preface **vii**

1 Preliminaries **1**
1.1 Words and Languages 1
1.2 K-adic Representation 2
1.3 Partial Functions 3
1.4 Graphs 4
1.5 Propositional Logic 6
1.5.1 Boolean Functions 8
1.6 Cardinality 8
1.6.1 Ordered Sets 10
1.7 Elementary Algebra 11
1.7.1 Rings and Fields 11
1.7.2 Groups 15
1.7.3 Number Theory 17

2 Introduction to Computability **22**
2.1 Turing Machines 23
2.2 Turing-Machine Concepts 26
2.3 Variations of Turing Machines 28
2.3.1 Multitape Turing Machines 29
2.3.2 Nondeterministic Turing Machines 31
2.4 Church's Thesis 34
2.5 RAMs 36
2.5.1 Turing Machines for RAMS 39

3 Undecidability **41**
3.1 Decision Problems 41
3.2 Undecidable Problems 43
3.3 Pairing Functions 46
3.4 Computably Enumerable Sets 47
3.5 Halting Problem, Reductions, and Complete Sets 50
3.5.1 Complete Problems 52
3.6 *S-m-n* Theorem 53
3.7 Recursion Theorem 55
3.8 Rice's Theorem 57
3.9 Turing Reductions and Oracle Turing Machines 59
3.10 Recursion Theorem, Continued 66
3.11 References 69
3.12 Additional Homework Problems 70

4 Introduction to Complexity Theory **72**
4.1 Complexity Classes and Complexity Measures 74
4.1.1 Computing Functions 76
4.2 Prerequisites 77

5 Basic Results of Complexity Theory **78**
5.1 Linear Compression and Speedup 80
5.2 Constructible Functions 86
5.2.1 Simultaneous Simulation 87
5.3 Tape Reduction 90
5.4 Inclusion Relationships 97
5.4.1 Relations between the Standard Classes 105
5.5 Separation Results 107
5.6 Translation Techniques and Padding 111
5.6.1 Tally Languages 113
5.7 Relations between the Standard Classes—Continued 115
5.7.1 Complements of Complexity Classes: The Immerman–Szelepcsényi Theorem 116
5.8 Additional Homework Problems 120

6 Nondeterminism and NP-Completeness **122**
6.1 Characterizing NP 123
6.2 The Class P 124
6.3 Enumerations 126
6.4 NP-Completeness 128
6.5 The Cook–Levin Theorem 130
6.6 More NP-Complete Problems 136
6.6.1 The Diagonal Set Is NP-Complete 137
6.6.2 Some Natural NP-Complete Problems 138

6.7 Additional Homework Problems 142

7 Relative Computability **145**
7.1 NP-Hardness 147
7.2 Search Problems 151
7.3 The Structure of NP 153
7.3.1 Composite Number and Graph Isomorphism 158
7.3.2 Reflection 161
7.4 The Polynomial Hierarchy 162
7.5 Complete Problems for Other Complexity Classes 170
7.5.1 PSPACE 170
7.5.2 Exponential Time 174
7.5.3 Polynomial Time and Logarithmic Space 175
7.5.4 A Note on Provably Intractable Problems 179
7.6 Additional Homework Problems 179

References **181**

Author Index **187**

Subject Index **191**