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Friedrich Hirzebruch   Topological Methods in Algebraic Geometry

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To my teachers

Heinrich Behnke and Heinz Hopf

Preface to the first edition

In recent years new topological methods, especially the theory of sheaves founded by J. LERAY, have been applied successfully to algebraic geometry and to the theory of functions of several complex variables. H. CARTAN and J.-P. SERRE have shown how fundamental theorems on holomorphically complete manifolds (STEIN manifolds) can be formulated in terms of sheaf theory. These theorems imply many facts of function theory because the domains of holomorphy are holomorphically complete. They can also be applied to algebraic geometry because the complement of a hyperplane section of an algebraic manifold is holomorphically complete. J.-P. SERRE has obtained important results on algebraic manifolds by these and other methods. Recently many of his results have been proved for algebraic varieties defined over a field of arbitrary characteristic. K. KODAIRA and D. C. SPENCER have also applied sheaf theory to algebraic geometry with great success. Their methods differ from those of SERRE in that they use techniques from differential geometry (harmonic integrals etc.) but do not make any use of the theory of STEIN manifolds. M. F. ATIYAH and W. V. D. HODGE have dealt successfully with problems on integrals of the second kind on algebraic manifolds with the help of sheaf theory.

I was able to work together with K. KODAIRA and D. C. SPENCER during a stay at the Institute for Advanced Study at Princeton from 1952 to 1954. My aim was to apply, alongside the theory of sheaves, the theory of characteristic classes and the new results of R. THOM on differentiable manifolds. In connection with the applications to algebraic geometry I studied the earlier research of J. A. TODD. During this time at the Institute I collaborated with A. BOREL, conducted a long correspondence with THOM and was able to see the correspondence of KODAIRA and SPENCER with SERRE. I thus received much stimulating help at Princeton and I wish to express my sincere thanks to A. BOREL, K. KODAIRA, J.-P. SERRE, D. C. SPENCER and R. THOM.

This book grew out of a manuscript which was intended for publication in a journal and which contained an exposition of the results obtained during my stay in Princeton. Professor F. K. SCHMIDT invited me to use it by writing a report for the "Ergebnisse der Mathematik". Large parts of the original manuscript have been taken over unchanged, while other parts of a more expository nature have been expanded. In this way the book has become a mixture between a report, a textbook and an original article. I wish to thank Professor F. K. SCHMIDT for his great interest in my work.

I must thank especially the Institute for Advanced Study at Princeton for the award of a scholarship which allowed me two years of undisturbed work in a particularly stimulating mathematical atmosphere. I wish to thank the University of Erlangen which gave me leave of absence during this period and which has supported me in every way; the Science Faculty of the University of Münster, especially Professor H. BEHNKE, for accepting this book as a Habilitationsschrift; and the Society for the Advancement of the University of Münster for financial help during the final preparation of the manuscript. I am indebted to R. REMMERT and G. SCHEJA for their help with the proofs, and to H.-J. NASTOLD for preparing the index. Last, but not least, I wish to thank the publishers who have generously complied with all my wishes.

Fine Hall, Princeton F. HIRZEBRUCH

23 January 1956

Preface to the third edition

In the ten years since the publication of the first edition, the main results have been extended in several directions. On the one hand the RIEMANN-ROCH theorem for algebraic manifolds has been generalised by GROTHENDIECK to a theorem on maps of projective algebraic varieties over a ground field of arbitrary characteristic. On the other hand ATIYAH and SINGER have proved an index theorem for elliptic differential operators on differentiable manifolds which includes, as a special case, the RIEMANN-ROCH theorem for arbitrary compact complex manifolds.

There has been a parallel development of the integrality theorems for characteristic classes. At first these were proved for differentiable manifolds by complicated deductions from the almost complex and algebraic cases. Now they can be deduced directly from theorems on maps of compact differentiable manifolds which are analogous to the RIEMANN-ROCH theorem of GROTHENDIECK. A basic tool is the ring $K(X)$ formed from the semi-ring of all isomorphism classes of complex vector bundles over a topological space X, together with the BOTT periodicity theorem which describes $K(X)$ when X is a sphere. The integrality theorems also follow from the ATIYAH-SINGER index theorem in the same way that the integrality of the TODD genus for algebraic manifolds follows from the RIEMANN-ROCH theorem.

Very recently ATIYAH and BOTT obtained fixed point theorems of the type first proved by LEFSCHETZ. A holomorphic map of a compact complex manifold V operates, under certain conditions, on the cohomology groups of V with coefficients in the sheaf of local holomorphic sections of a complex analytic vector bundle W over V. For a special class of holomorphic maps, ATIYAH and BOTT express the alternating sum of the traces of these operations in terms of the fixed point set of the map. For the identity map this reduces to the RIEMANN-ROCH theorem. Another application yields the formulae of LANGLANDS (see 22.3) for the dimensions of spaces of automorphic forms. ATIYAH and BOTT carry out these investigations for arbitrary elliptic operators and differentiable maps, obtaining a trace formula which generalises the index theorem. Their results have a topological counterpart which generalises the integrality theorems.

The aim of the translation has been to take account of these developments – especially those which directly involve the TODD genus – within the framework of the original text. The translator has done this chiefly by the addition of bibliographical notes to each chapter and by a new appendix containing a survey, mostly without proofs, of some of the applications and generalisations of the RIEMANN-ROCH theorem made since 1956. The fixed point theorems of ATIYAH and BOTT could be mentioned only very briefly, since they became known after the manuscript for the appendix had been finished. A second appendix consists of a paper by A. BOREL which was quoted in the first edition but which has not previously been published. Certain amendments to the text have been made in order to increase the usefulness of the book as a work of reference. Except for Theorems 2.8.4, 2.9.2, 2.11.2, 4.11.1–4.11.4, 10.1.1, 16.2.1 and 16.2.2 in the new text, all theorems are numbered as in the first edition.

The author thanks R. L. E. SCHWARZENBERGER for his efficient work in translating and editing this new edition, and for writing the new appendix, and A. BOREL for allowing his paper to be added to the book.

We are also grateful to Professor F. K. SCHMIDT for suggesting that this edition should appear in the "Grundlehren der mathematischen Wissenschaften", to D. ARLT, E. BRIESKORN and K. H. MAYER for checking the manuscript, and to ANN GARFIELD for preparing the typescript. Finally we wish to thank the publishers for their continued cooperation.

Bonn and Coventry F. HIRZEBRUCH
23 January 1966 R. L. E. SCHWARZENBERGER

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